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ABSTRACT

The Van Hiele level theory offers an explanation and a remedy for student difficulty with higher-order cognitive processes required for success in secondary school geometry. This document reports results of a study which involved about 2700 students in 13 high schools, selected to provide broad representation of community socio-economics in the United States. The investigation looked at: (1) How are entering geometry students distributed with respect to the levels in the Van Hiele scheme; (2) What changes in Van Hiele levels take place after a year's study of geometry; (3) To what extent are levels related to concurrent geometry achievement; (4) To what extent do levels predict geometry achievement after a year's study; (5) What generalizations can be made concerning the entering Van Hiele level and geometry knowledge of students who are later found to be unsuccessful in their study of geometry; (6) To what extent is geometry being taught to students appropriate to their level; and (7) To what extent do geometry classes in different schools and socio-economic settings differ in content appropriateness to student level. (MP)

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VAN HIELE LEVELS AND ACHIEVEMENT
IN
SECONDARY SCHOOL GEOMETRY

CDASSG PROJECT

THE UNIVERSITY OF CHICAGO

SE 038813

VAN HIELE LEVELS AND ACHIEVEMENT
IN
SECONDARY SCHOOL GEOMETRY

by

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Mary Grace Kantowski, University of Florida
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All statements and interpretations presented herein are the author's and do not mean to suppose or suggest policies or beliefs of anyone else associated with the project or the National Institute of Education.

The CDASSG Project was conceived by Zalman Usiskin and Sharon Senk.

Coding of data was done under the direction of Roberta Dees; computer programs to analyze the data were written and run by Roberta Dees and Sharon Senk.

The author of this paper was responsible for the overall direction of the project, sample selection, choice and construction of tests, and selection of analyses.

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VAN HIELE LEVELS AND ACHIEVEMENT
IN
SECONDARY SCHOOL GEOMETRY

CHAPTER I: INTRODUCTION

In the United States, secondary school geometry is usually studied in a single year, normally in the tenth grade. The approach used in geometry tends to be different than the approach taken in other secondary school mathematics courses, in that the student is introduced to the workings of a mathematical system (through experiences with postulates, theorems, definitions, and practice with proof) at the same time that the student is learning the content of the course. Thus, though the course assumes very little prior content knowledge, it is taught quite abstractly.

The van Hiele level theory, developed by two Dutch mathematics educators in the late 1950's, has been applied to explain why many students have difficulty with the higher order cognitive processes, particularly proof, required for success in high school geometry. It has been theorized that students who have trouble are being taught at a higher van Hiele level than they are at or ready for. The theory also offers a remedy: go through the sequence of levels in a specific way.

The CDASSG project was designed to address a variety of questions relating to this theory.

1. How are entering geometry students distributed with respect to the levels in the van Hiele scheme?

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2. What changes in van Hiele levels take place after a year's study of geometry?
3. To what extent are van Hiele levels related to concurrent geometry achievement?
4. To what extent do van Hiele levels predict geometry achievement after a year's study?
5. What generalizations can be made concerning the entering van Hiele level and geometry knowledge of students who are later found to be unsuccessful in their study of geometry?
6. To what extent is the geometry being taught to students appropriate to their van Hiele level?
7. To what extent do geometry classes in different schools and socio-economic settings differ in the appropriateness of the content to the van Hiele level of the student?

The CDASSG project was given three years to study these questions, a year (1979-80) to design the study, a year (1980-81) to carry out the study, and a year (1981-82) to analyze the data and write reports.

The study involved, in toto, about 2700 students in 13 high schools selected from throughout the United States to provide a broad representation of community socio-economics. These students were given tests of geometry knowledge and a van Hiele level assessment at the beginning and end of the school year. Proof achievement of a major portion of the sample was also assessed at the end of the school year. A variety of other data was collected.

This paper is the report of the results of the study.

CHAPTER II: THE VAN HIELE LEVEL THEORY

What has become known as the van Hiele level theory was developed by Dina van Hiele-Geldof and her husband Pierre Marie van Hiele in separate doctoral dissertations at the University of Utrecht in 1957. Dina died shortly after her dissertation was completed; Pierre has thus been the one to explicate the theory. In the years 1958-59 he wrote three papers (two in English, one in Dutch but translated into French) that received little attention in the West, but were applied in curriculum development by the Soviet academician Pyshkalo (1968). Freudenthal, the van Hieles' mentor, publicized the theory in his well-known book Mathematics as an Educational Task (1973). Through Freudenthal and the Soviets, the work of the van Hieles came to the attention of Wirszup, who was the first to speak about the van Hiele theory on this side of the Atlantic (1974) and later published his speech (1976).

Wirszup's paper spawned a variety of recent efforts. Hoffer, who had written a secondary school geometry text (1979) in which much time is devoted to preparing for proof, visited with van Hiele in the Netherlands, found a similar thinker, and wrote about the levels (1981). Two projects other than this one dealing with aspects of the theory were funded (Burger, 1981; Geddes, 1981), and at least one dissertation testing an aspect of the theory has been completed (Mayberry, 1981). A second dissertation uses the same data as in this paper and should be consulted by anyone interested in the results presented here (Senk, in preparation).

The theory has three aspects: the existence of levels, properties of the levels, and the movement from one level to the next.

Existence of levels. According to the theory, there are five levels of understanding in geometry. These levels are described by the van Hieles in various places in both general and behavioral terms. Summary general descriptions and examples from Hoffer (1979, 1981) are given here; the names in parentheses are his.

Level 1:	(recognition) The student can learn names of figures and recognizes a shape as a whole. (Squares and rectangles seem to be different.)
Level 2:	(analysis) The student can identify properties of figures. (Rectangles have four right angles.)
Level 3:	(order) The student can logically order figures and relationships, but does not operate within a mathematical system. (Simple deduction can be followed, but proof is not understood.)
Level 4:	(deduction) The student understands the significance of deduction and the roles of postulates, theorems, and proof. (Proofs can be written with understanding.)
Level 5:	(rigor) The student understands the necessity for rigor and is able to make abstract deductions. (Non-Euclidean geometry can be understood.)

The van Hieles number these levels 0 through 4, not 1 through 5, and Dina called levels 2-5, respectively, the aspect of geometry, the essence of geometry, insight into the theory of geometry, and scientific insight into geometry (van Hiele-Geldof, 1957).

Properties of levels. It is inherent in the van Hiele theory that, in understanding geometry, a person must go through the levels in order. We call this the fixed sequence property of the levels.

Property 1: (fixed sequence) A student cannot be at van Hiele level n without having gone through level n-1.

P.M. van Hiele (1958-59) identifies other properties of the levels, to which we have assigned names.

Property 2: (adjacency) At each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level.

Property 3: (distinction) Each level has its own linguistic symbols and its own network of relationships connecting those symbols.

Property 4: (separation) Two persons who reason at different levels cannot understand each other.

To exemplify these properties, consider the student who remarks to a geometry teacher, "I can follow a proof when you do it in class, but I can't do it at home.". This student may be at level 3 while the teacher is operating at level 4. Property 4 indicates that the student cannot understand the teacher, and Property 3 explains why there is no understanding, for the teacher is using objects (propositions, in the case of proof) and a network of relationships (proof itself) which the student does not yet understand used in this way. If the student is at level 3, then the student's network consists of simple ordering of propositions, and Property 2 indicates that these orderings, intrinsic at level 3, become extrinsic at level 4.

Movement from one level to the next. Van Hiele (1959) is more optimistic than Piaget, believeing that cognitive development in geometry can be accelerated by instruction. The van Hieles (P.M. and Dina, 1958; P.M., 1959) have given detailed explanations of how the teacher should operate to lead students from one level to the next. We consider this specification as a fifth property of the levels.

Property 5: (attainment) The learning process leading to complete understanding at the next higher level has five phases, approximately but not strictly sequential, entitled:

inquiry
directed orientation
explanation
free orientation
integration

It is not the intent of this study to examine the movement from one level to the next. The interested reader may wish to look at Hoffer (1982). The writings of the van Hieles serve to indicate that the process of moving from one level to the next takes more time than can be spanned in an hour or even a short unit of teaching. For instance, Dina (1957) reports 20 lessons to get from level 1 to level 2 (our numbering) and 50 lessons to get from level 2 to level 3, working with 12-year-olds. This is about a half year of lessons if studied continuously.

Properties of the theory. From the descriptions of the van Hiele theory given thus far, the reader may have noted that this theory possesses three appealing characteristics: elegance, comprehensiveness, and wide applicability. By elegance we mean that the theory involves a rather simple structure described by reasonably succinct statements, each with broad effect. For instance, the same principles apply for movement from level 1 to 2 as from 2 to 3 and so on, displaying an elegance of form. And the simplicity of structure is evident when one notes that the figures of level 1 are the building blocks for properties at level 2, which in turn are ordered in level 3, the ordering being an essential prerequisite for the understanding of a mathematical system at level 4, one of those objects compared at level 5.

Any theory which covers the whole of learning of geometry, and which seeks to explain not only why students have trouble in learning but also what could be done to remove these stumbling blocks, must be called comprehensive. P.M. van Hiele asserts in Begrip en Inzicht that the theory applies to all of mathematical understanding and gives examples involving the learning of functions and other non-geometric notions. Yet the theory has not been detailed enough in other areas to make it that comprehensive. From personal communication, we know that Mayberry felt restricted by the lack of breadth even of geometric content in the published articles of van Hiele. For this same reason, the study of Burger et al. (1981) has restricted the domain to triangles and quadrilaterals. Still, the theory purports to be quite comprehensive.

With attempts to apply the theory in geometry curricula in countries as diverse as the Netherlands, the Soviet Union, and the United States, the theory is obviously seen as both widely and easily applicable.

Significantly, these properties of a theory (elegance, comprehensiveness, and wide applicability) do not lend themselves to being tested. Yet they are probably the major reasons for the speed with which the van Hiele theory has become known in the United States. Thus many mathematics educators are accepting and using this theory on the basis of characteristics of the theory rather than a testing of its individual components.

An analogous situation would be if someone had a theory for curing all cancer, requiring merely the introduction of a single substance into the bloodstream in some carefully administered way. That would be an elegant cure, and comprehensive in the sense that it applied to all cancers. If it were accepted in many places, the theory would have gained the quality of wide applicability. Indeed there exists such a theory, and the substance is laetrile. Yet laetrile has not withstood more careful scrutiny. Just because a theory is elegant, comprehensive, and has been used by many does not insure that the theory is correct.

In the medical situation, one looks for experiments that test the ability of the theory to satisfy its claims. The van Hiele theory includes descriptions of behaviors of students at various levels and predicts certain other behaviors of those students. Descriptive accuracy and predictive power are important attributes of theories that purport to be scientific (as opposed to theories that are only speculative). The fundamental purpose of this project is to test the ability of the van Hiele theory to describe and predict the performance of students in secondary school geometry. Referring back to the questions to be addressed by the project, as stated on pages 1 and 2 of this report, the first two questions are each designed to test the extent to which a level can be identified for each student and to test the fixed sequence property of the levels. Questions 3, 4, and 5 test the ability of van Hiele levels to predict geometry performance. Questions 6 and 7 relate to the separation property of the levels and provide a somewhat less formal test of the validity of that property.

CHAPTER III: BEHAVIORS AT EACH VAN HIELE LEVEL

In order to be subjected to a rigorous test, a theory must be described in sufficient detail and clarity to enable test instruments to be devised. For the van Hiele theory, this means that the levels must be very accurately identified.

Accordingly, in late 1979 and early 1980, all of the van Hieles' writings available to the CDASSG project personnel were examined for quotes that described behaviors of students at a given level. A total of nine works were examined, four originally written in English, five translated into English from Dutch, German, or French. The following is a list of behaviors, sorted by level.

Level 1 (their base level, level 0)

(P.M., 1958-59)

1. "Figures are judged according to their appearance."
2. "A child recognizes a rectangle by its form, shape.
3. . . . and the rectangle seems different to him from a square."
4. "When one has shown to a child of six, a six year old child, what a rhombus is, what a rectangle is, what a square is, what a parallelogram is, he is able to produce those figures without error on a geoboard of Gattegno, even in difficult situations."
5. "a child does not recognize a parallelogram in a rhombus."
6. "the rhombus is not a parallelogram. The rhombus appears. . . as something quite different."

(P.M., 1968)

7. "when one says that one calls a quadrilateral whose four sides are equal a rhombus this statement will not be enough to convince the beginning student [from which I deduce that this is his level 0] that the parallelograms which he calls squares are part of the set of rhombuses."

(P.M., 1979)

8. (on a question involving recognition of a tilted square as a square) "basic level, because you can see it!"

Level 2 (their first level)

(P.M., 1957)

1. "He is able to associate the name 'isosceles triangle' with a specific triangle, knowing that two of its sides are equal, and draw the subsequent conclusion that the two corresponding angles are equal."

(Dina, 1957; P.M. and Dina, 1958)

2. ". . . a pupil who knows the properties of the rhombus and can name them, will also have a basic understanding of the isosceles triangle = semirhombus."
3. "The figures are the supports (lit. 'supports' in French) of their properties."
4. "That a figure is a rectangle signifies that it has four right angles, it is a rectangle, even if the figure is not traced very carefully."
5. "The figures are identified by their properties. (E.g.) If one is told that the figure traced on the blackboard possesses four right angles, it is a rectangle, even if the figure is not traced very carefully."
6. "The properties are not yet organized in such a way that a square is identified as being a rectangle."

(P.M., 1959)

7. "The child learns to see the rhombus as an equilateral quadrangle with identical opposed angles and interperpendicular diagonals that bisect both each other and the angles."
8. (a middleground between this and the next level) "Once the child gets to the stage where it knows the rhombus and recognizes the isoceles triangle for a semi-rhombus, it will also be able to determine offhand a certain number of properties of the equilateral triangle."
9. "Once it has been decided that a structure is an 'isosceles triangle' the child will also know that a certain number of governing properties must be present, without having to memorize them in this special case."

(P.M., 1976)

10. "The inverse of a function still belongs to the first thought level."
11. "Resemblance, rules of probability, powers, equations, functions, revelations, sets - with these you can go from zero to the first thought level."

Level 3 (their second level)

(Dina, 1957)

1. "Pupils . . . can understand what is meant by 'proof' in geometry. They have arrived at the second level of thinking."

(P.M., 1957)

2. "He can manipulate the interrelatedness of the characteristics of geometric patterns."
3. "e.g., if on the strength of general congruence theorems, he is able to deduce the equality of angles or linear segments of specific figures."

(P.M., 1958-59)

4. "The properties are ordered [lit. 'ordonnent']. They are deduced from each other: one property precedes or follows another property."
5. "The intrinsic significance of deduction is not understood by the student."
6. "The square is recognized as being a rectangle because at this level definitions of figures come into play."

(P.M., 1959)

7. "the child . . . [will] recognize the rhombus by means of certain of its properties, . . . because , e.g., it is a quadrangle whose diagonals bisect each other perpendicularly."
8. "It [the child] is not capable of studying geometry in the strictest sense of the word."
9. "The child knows how to reason in accordance with a deductive logical system . . . this is not however, identical with reasoning on the strength of formal logic."

(P.M., 1976)

10. "the question about whether the inverse of a function is a function belongs to the second thought level."
11. "The understanding of implication, equivalence, negation of an implication belongs to the second thought level."

(P.M., 1978)

12. "they are able to understand more advanced thought structure, such as: 'the parallelism of the lines implies (according to their signal character) the presence of a saw, and therefore (according to their symbolic character) equality of the alternate-interior angles'."
13. "I [the student] can learn a definition by heart. No level. I can understand that definitions may be necessary: second level."
14. ". . . you know what is meant by it [the use of 'some' and 'all'] second level."

Level 4 (their third level)

(P.M., 1957)

1. "He will reach the third level of thinking when he starts manipulating the intrinsic characteristics of relations. For example: if he can distinguish between a proposition and its reverse" [sic, meaning our converse]

(Dina, 1957)

2. "we can start studying a deductive system of propositions, i.e, the way in which the interdependency of relations is effected. Definitions and propositions now come within the pupils' intellectual horizon."
3. "Parallelism of the lines implies equality of the corresponding angles and vice versa."

(P.M. and Dina, 1958)

4. "The pupil will be able, e.g., to distinguish between a proposition and its converse."
5. "it (is) . . . possible to develop an axiomatic system of geometry".

(P.M., 1958-59)

6. "The mind is occupied with the significance of deduction, of the converse of a theorem, of an axiom, of the conditions necessary and sufficient."

(P.M., 1968)

7. ". . . one could tell him (the student) that in a proof it is really a question of knowing whether these theses are true or not, or rather of the relationship between the truth of these theses and of some others. Without their understanding such relationships we cannot explain to the student that one has to have recourse to axioms." [I induced the level from the first part of this statement; he never identifies the level.]

Level 5 (their fourth level)

(Dina, 1957)

1. "A comparative study of the various deductive systems within the field of geometrical relations is . . . reserved for those, who have reached the fourth level . . . ".

(P.M. and Dina, 1958)

2. "finally at the fourth level (hardly attainable in secondary teaching) logical thinking itself can become a subject matter."
3. " the axiomatics themselves belong to the fourth level."

(P.M., 1958-59)

4. "one doesn't ask such questions as: what are points, lines, surfaces, etc.? . . . Figures are defined only by symbols connected by relationships. To find the specific meaning of the symbols, one must turn to lower levels where the specific meaning of these symbols can be seen."

Our bibliography for the van Hieles was not complete at the time these descriptions were gathered. We did not have all of the workbooks (1978) or, more significantly, the book Begrip en inzicht (1973) (Understanding and Insight, 1979). We did not have either P.M.'s or Dina's complete dissertation (1957). Since that time we have examined these works and found that they add little to the details presented here.

There is a paucity of behaviors at level 5, and even those four behaviors listed are quite vague. For instance, the second behavior listed mentions "logical thinking itself" as a subject matter. One person might interpret this statement to refer to axiomatics (as the third behavior listed suggests) or to symbolic logic (which is more common in classrooms).

A variety of behaviors is described for level 4, but the descriptions are often vague. For example, the sixth behavior at that level depends upon the meaning of the words "occupied" and "significance". Though a teacher can, in the course of a typical year's study of geometry, identify a number of statements from students which seem to exemplify this behavior, a situation which tests whether occupation or significance could be exhibited is not immediately apparent.

At levels 1, 2, and 3 the behaviors are in sufficient quantity and detail to enable testing Thus we concluded that the van Hiele principles are easily testable at the first three levels that, with some effort level 4 could be tested, but that level 5 is of questionable testability. This did not dissuade us from constructing a test covering all five levels, but our doing so was done with the knowledge that disagreement with our level 5 questions was likely and that all conclusions regarding level 5 would be subject to this overall caveat.

In this conclusion, we were supported by P.M. van Hiele himself. During a trip through the United States in 1980, van Hiele disavowed belief in a fifth level, thus changing a view he had expressed some years before (P.M., 1959-2), but reflecting questioning of this level as reported in Begrip en Inzicht. Hoffer reported to us (in personal communication) that on this trip he had to re-convince P.M. of the existence of the second highest level (our level 4)! P.M. made no mention of either of these higher levels in his invited address to the AERA-NCTM Research Presession in Seattle (1980).

Removing one level from the theory would not be disastrous to it. But removing two levels results in a theory that surely would not have been as attractive to the mathematics education community because it would not so clearly locate proof understanding and would, with three levels, be seen as too simplistic.

Late in 1980, Linda Sears, a University of Chicago student, noted a resemblance between the levels of development of the van Hieles and the approach Maria Montessori takes in geometry. She wrote a master's paper comparing the theories (1981). During that writing she learned from Dorothy Geddes that P.M. and Dina had been Montessori teachers for seven years. This connection is not acknowledged in any of the writings of the van Hieles. However, the influences of Gestalt theory and Otto Selz are acknowledged by both van Hieles (P.M., 1957; Dina, 1957). Ideas of Mannoury and others were also accommodated into the theory (Sears, 1981; see also Dina, 1957).

CHAPTER IV: DESIGN OF THE STUDY

Population. The population for this study consists of all studentsin the United States enrolled in a one-year geometry course. This course is taken by about 56% of males, 55% of females) who are or will become seniors in high school (Peng et al., 1981), and 51% of 17-year-olds have taken it (NAEP, 1979).

Sample. The sample studied consists of 2699 students enrolled in a one-year geometry course in 13 schools. Schools were selected on the basis of meeting certain socio-economic criteria and offering a high probability of success in obtaining reliable data consistent with the testing requirements. All one-year geometry classes in these schools were tested, and the only way that a student in these classes would not be in the sample is if the student refused to participate in the study* or was absent on all of the testing days.

Table 1 describes the schools and tracks within schools, and gives the number of classes and students in each school. All schools were in the home states of project staff, and the schools were selected to provide a representation of sizes of schools and education conditions nationwide. One school is private. Blacks are a majority in one school and sizeable minorities in two others; one school is primarily Hispanic; still another has a large Oriental population. Of the 2699 in the total sample, 1392 (51.6%) are male, 1307 (48.8%) are female. This is very close to the nationwide percentages of 50.8% and 49.2% for the 14-17 age group and 51.3% and 48.7% for the school population (Statistical Abstract of the U.S., 1980).

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*In accordance with government and University policy, in the testing at the beginning of the school year, students were given the option to participate or not to participate in the study. Nonparticipation ranged from 0% in seven schools to 13% of students in one school, and totalled less than 3% of the sample. In the spring testing, no such option was given so that at least one picture of the entire population could be provided. Still, a small percentage of students (less than one-half of one percent) used obvious aliases or exhibited other behavior indicating that they did not take the study seriously and these test papers were not used in any analyses.

Though secondary school geometry is traditionally taught in the tenth grade, and was a tenth grade course for the plurality of students in each of the schools in the study, only 56% of the total sample was in the tenth grade, and there were students from seventh through twelfth grades represented (Table 2). Ages of students ranged from 11 to 20 with 96% of students being between the ages of 14 and 17.

Overall design. A standard pretest, posttest design was used, involving four tests given to all students and one of three forms of a proof test given to some students, in accordance with the following schedule:

First week of school:	Entering Geometry Test (EG) Van Hiele Level Test (VHF)
Three to five weeks before end of school:	Van Hiele Level Test (VHS) Comprehensive Assessment Program Geometry Test (CAP) Proof Test (Prf)

The fall testing was done on consecutive days; the spring testing on three days within a five-day period. The Van Hiele Level tests given in fall and spring were identical.

The proof test was administered only to these classes that had studied proof and whose teacher gave permission for the testing. Of 99 classes involved in this study, 14 (all from the lowest track in 3-track schools) did not study proof and teachers of 10 others refused permission (Table 1).

The number of students in each test subsample is: EG, 2410; VHF, 2361; VHS, 2361; CAP, 2015; Prf1, 506; Prf2, 506; Prf3, 508; all of first 4 tests, 1596; all 5 tests, 1130.

Test construction. Of the four tests used in the study, two (VH and Prf) were constructed from scratch by the project, one (EG) was adapted from a test used elsewhere, and one (CAP) was selected from available standardized tests.

The EG test was designed to measure incoming geometry knowledge of students. The goal was to have a test which (1) covered content that an entering student who had encountered geometry in junior high school might know and (2) was relatively independent of van Hiele level ideas. Further, it was hoped to use a test in which there was very little project input, to minimize potential bias.

About a decade previous to this study, a student at Ohio State University (--- MacDonald, first name unknown to the project) had undertaken a study of geometry knowlege of entering geometry students. The multiple choice instrument used had 50 questions and project personnel had access to the percentage of students answering each choice on each question. A pilot test for the present study used the 20 easiest items on this earlier test, each question having been answered correctly by at least 45% of the sample a decade ago.

The only changes made in the items were in the foils (when a foil seemed misleading or was not particularly useful) and in the wording of stems. Piloting in three schools indicated that 35 minutes was more than adequate time, and that the items were not so difficult so as to discourage students with very little background.

During the fall testing, an error was discovered in the drawing for Question 14 and, as a result, all analyses were done utilizing a 19-item test. The test is Appendix A.

The van Hiele test (VHF and VHS) was designed to determine, if such a determination would be possible, the van Hiele level of the student. From quotes of the van Hieles themselves regarding student behaviors to be expected at each level, questions were written for each level that would test whether a student was at that level. The test construction and piloting are detailed in Porter (1980).

In a first piloting, the questions were given individually to students in oral interviews by three project personnel in three different states. Utilizing the responses of students, a 25-item multiple-choice test with 5 items at each level was created. This test was piloted with entire classes in four schools to ensure that it would not be too long for a 35-minute time limit.

Items were rejected or modified only if the responses of students did not seem to reflect the appropriate van Hiele level. Ease or difficulty of an item was never a criterion, though the goal was to have easy items at each van Hiele level. The van Hiele tests used in the study were essentially the same as those in the second piloting. The van Hiele test, with corresponding reference quotes, is Appendix B.

Only two commercially available standardized tests were felt to represent the breadth of content and language studied in geometry courses today: the Cooperative Test - Geometry published by the Educational Testing Service and the Comprehensive Assessment Program Geometry test published by Scott, Foresman and Co. The former test was used in some of the test site schools so was felt to have possible bias because teachers knew the items. The latter test was selected for use in this study.

The reader may well be wondering why the project needed to construct its own van Hiele level test. It might seem that questions on a standardized test could be sorted into levels and students given that test and their responses analyzed by levels. However, not every geometry question one asks is assignable to a van Hiele level. P.M. van Hiele's response (1979) to items that involve parroting a definition or theorem or applying a theorem in a numerical problem is usually "No level". When one analyzes a standard content test such as the CAP one finds very few questions that are not of this type. The CAP test contains perhaps six items classifiable into van Hiele levels: Item 11 (level 3), item 13 (level 4), item 25 (level 4), item 31 (level 2), item 35 (level 4), and item 37 (level 4). (Others might disagree with these choices and with the level designations.) The questions on the van Hiele level test are generally more conceptual than those found on standard exams and even the low level items require some sort of analysis. At level 1, one asks whether a drawing fits one's conception of a member of a class of figures. At level 2, one wonders whether a property is true always, not merely in a single figure. At level 3, one orders properties, needing to know whether one statement always follows from another. The CAP test is overwhelmingly constituted of items that do not possess the attributes of van Hiele level questions. Thus in theory the CAP test is rather independent of van Hiele level and a person might score high on it despite being at a low van Hiele level (through memorizing without understanding, van Hiele would say) and vice-versa (if the person were conceptually strong but weak in details).

The ability to write proofs is a different matter. Though van Hiele recognizes that a student can memorize how to do very simple proofs, it is claimed (Wirszup (1976), Hoffer (1981)) that a student must be at level 4 in order to understand proofs.

The proof tests were designed under the direction of Sharon Senk. The goal was to arrive at a measure of proof-writing achievement. In order to maximize the number of items with which proof-writing achievement could be measured, three non-overlapping test forms were created, each consisting of six items: one fill-in of statements or reasons in an almost-completed proof; one requiring a picture, the "given" and the "to prove" for a theorem stated in words; and four complete proofs. There was extensive piloting of items (Senk, 1982). Pilot testing indicated that each of the forms could be completed in 35 minutes.

The eighteen proof test items cover material common to the first two-thirds of the most widely-used geometry texts: congruent triangles, parallel lines, quadrilaterals and similarity. No proofs were given requiring knowlege of area, volume, or properties of circles. Of the four full proofs on each form, one was a proof of a standard textbook theorem (e.g., the diagonals of a rectangle are congruent) and the other three were like the proof exercises found in standard texts. The complete proof tests form Appendix C.

The tests were intended to be approximately equal in difficulty but not designed to be statistically equivalent. (This was fortunate: Results show them to be not statistically equivalent.) Each was designed to be similar to a test of proof a geometry teacher might give. The cover pages of the three proof tests were identical so that students and teachers would not know which form was being distributed without looking inside. The project distinguished the forms by numbering them consecutively in such a way that all copies of form n had an identification number congruent to n modulo 3.

Test administration. All tests were administered by classroom teachers and monitored by project personnel. That is, a project representative was situated in each school on each day of testing and usually was present in the geometry classroom during the time of testing.

Scripts were provided by the project for each test and it was the rule, rather than the exception, that these scripts were followed to the letter. The test scripts constitute Appendix D.

While students were taking the tests, monitors counted the numbers of students in each classroom and the number taking the test. Monitor forms constitute Appendix E. During the spring testing, teachers were asked to complete forms relating to the content of the course and to verify the class list of students obtained from the fall testing. Teacher-completed forms contitute Appendix F.

Test grading. Student responses from the EG, VHF, VHS, and CAP tests were transcribed into computer storage. After the numerical score was determined for each proof response, these scores were also transcribed. The accuracy of all transcriptions was independently verified. Grading of the tests, as well as all analyses, was done using release 79.5 of the Statistical Analysis System (SAS) on the Amdahl computer at the University of Chicago.

For the van Hiele tests, a student was assigned a weighted sum score in the following manner:

1 point for meeting criterion on items 1-5 (Level 1)
2 points for meeting criterion on items 6-10 (Level 2)
4 points for meeting criterion on items 11-15 (Level 3)
8 points for meeting criterion on items 16-20 (Level 4)
16 points for meeting criterion on items 21-25 (Level 5)

The criterion used was either 3 of 5 correct or 4 of 5 correct (the latter being called the stricter criterion).* The points were added to give the weighted sum. The method by which the weighted sum was calculated allows a person to determine upon which levels the criterion has been reached from the weighted sum alone. For example, a score of 19 indicates that the student reached the criterion on levels 1, 2, and 5. In this way, a single number from 0 to 31 is equivalent to having 5 separate Yes-No decisions on the 5 levels.

The proof tests were graded under a scheme roughly following that used in Advanced Placement examinations. Eight high school mathematics teachers were hired for a week (at $70/day) to grade all papers. The grading was blind; graders saw no names, only six digit IDs. A different pair independently read each item on a student's test and graded the response from 0 to 4. Criteria for grading each item were discussed by the teachers before grading of that item began; this resulted in 86% agreement within the pairs. Readers disagreed by more than one point on less than 2% of items. In case of disagreement, a third reader graded the item (again independently) and the median score was given the student for that item.

To our knowledge, and after an in-depth perusal of the literature,

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*The choice of criterion, given the nature of this test, is based upon whether one wishes to reduce Type I or Type II error. Recall that Type I error refers to a decision made (in this case a student meeting a criterion) when it should not have been made.

P(3 of 5 correct by random guessing) = .05792

P(4 of 5 correct by random guessing) = .00672

So the 4 of 5 criterion avoids about 5% of cases in which Type I error may be expected to manifest itself. However, consider the probability of Type II error, the probability that a student who is operating at a given level at, let's say, 90% mastery, a rather this resulted in the largest assessment of geometry proof-writing ability every undertaken in the United States. A proof-writing assessment on this scale has not been done previously due, it seems, to perceived problems in obtaining agreement among graders and in finding items that could be fairly given to such a broad student population.

We have already mentioned that the items were selected to represent the proofs that might be found in the first two-thirds of the year of geometry. Finding items was not any more difficult than in any other test of achievement; we were able to select from a reasonably large item pool.

The scale from 0 to 4 used to grade the items was based upon a scale suggested by Malone et al. (1980) and modified by Sharon Senk for use with geometric proofs. The scale and the modifications are found along with the three forms of the test in Appendix C. A measure of the wide applicability of the grading procedures was that one of the schools taught students to write all proofs in "flow proof" form (Allen and Guyer, 1973). Once the graders were aware of the conventions used in such proofs, this resulted in no special problems.

(continued from previous page)

strong criterion, will be found by the test to not meet the criterion.

P(less than 3 of 5 correct given 90% chance on each item) = .00856

P(less than 4 of 5 correct given 90% chance on each item) = .08146

These are of the same orders of magnitude, in the other direction, as the probabilities associated with Type I error. The 3 of 5 criterion avoids about 7% of cases in which Type II error may be expected to appear. If weaker mastery, say 80%, is expected of a student operating at a given level, then it is absolutely necessary to use the 3 of 5 criterion, for Type II errors with the stricter criterion are much too frequent.

P(less than 3 of 5 correct given 80% chance on each item) = .05792

P(less than 4 of 5 correct given 80% chance on each item) = .26272

We could not have been happier with our grading procedures. During the week of grading, the eight graders developed an esprit de corps which led them to want to agree and caused them to look upon their task with surprising zest. We encourage others to use the 0 to 4 scheme and to follow our other procedures (see Senk (1982) for more detail).

Operational definitions. The grading gave rise to the following operational definitions.

1. Entering geometry knowledge: number of items correct (19 maximum) on EG test

2. Classical van Hiele level (i.e., the level if the entire theory is considered):

Level*	corresponds to	Weighted sum
0		0
1		1
2		3
3		7
4		15
5		31

3. Modified van Hiele level (the level if level 5 is excluded from consideration):

Level*	corresponds to	Weighted sum
0		0 or 16
1		1 or 17
2		3 or 19
3		7 or 23
4		15 or 31

---

*The assigning of levels in either the classical or modified case requires that a student's responses satisfy Property 1 of the levels, i.e., that the student at level n satisfy the criterion not only at that level but also at all preceding levels. Thus a student who satisfies the criterion at levels 1, 2 and 5, for instance, would have a weighted sum of 1 + 2 + 16 or 19 points, would have no classical van Hiele level, but would be assigned the modified van Hiele level 2. A student who satisfies the criterion at level 3 only would not be assigned either a classical or modified van Hiele level. Neither of these students would be said to fit the classical van Hiele model. (One key question regards the percentage of students that do fit the model.)

4. Geometry achievement after a year's study:
    - A. Knowledge of standard content: number of items correct (40 maximum) on CAP test
    - B. Proof-writing ability: total score on Prf test (24 maximum), called PrfTOT,

      or

      number of proofs (items 3-6) upon which student scored 3 or 4 points, called PrfCOR.

5. Unsuccessful student
    - A. One who scores below 13 on the CAP test
    - B. One who scores below 3 on every one of the four proofs on the Prf test (PrfCOR = 0)

The forms completed by the teachers or by a representative of the school gave rise to one other definition.

6. Geometry class with proof emphasis: Any class in a school that does not track: either track of a school with two tracks: the two higher tracks of a school with three tracks.

The socio-economics of schools were determined subjectively by the member of the project Advisory Board in that school's area.

CHAPTER V: STUDENT PARTICIPATION AND TEST RELIABILITY

Unlike many studies that test only a couple of classes from a given school, the attempt was made here to test all students in one-year geometry classes in the schools involved in the study, so as to maximize the likelihood that a student tested in the fall would also be tested in the spring. Yet of the 2699 students who took at least one of the tests, only 1596 (the ALL4 sample) took all four of the EG, VHF, VHS, and CAP tests. The loss of students occured as follows:

> Of the 2699 in the total sample, 2487 (92%) were present on the first test day and 2410 (97%) took EG.
>
> Of the 2410 who took EG, 2285 (95%) took VHF.
>
> Of the 2285 who took EG and VHF, 1765 (77%) took VHS.
>
> Of these 1765, 1596 (90%) took CAP.

As one would expect, the largest dropoff comes between the fall and spring testing. From previous studies, this dropoff was anticipated, and an effort was made to determine the situations of those students who were present for the fall testing but not for the spring--did they withdraw failing, withdraw passing, leave school, or were they merely absent on the testing days?

Teachers were asked to check their spring class lists against those in the fall and provide a corrected list categorizing those students who had left into one of the four categories mentioned above. The data collected turned out to be quite unreliable. A teacher would classify a student as having left school but we would find the student in a different class. A student would be classified as having withdrawn passing into a different geometry class but we would find no record of the student in any other class. Thus we cannot provide a picture of the general reasons why students might leave geometry and, more importantly for the present study, we could not do planned analyses of the extent to which EG or VHF scores might be used to foresee a potential for nonsurvival in the geometry course.

Still, from the data we have, a picture of striking differences between schools emerges. A rough determination of holding power in the geometry class is the ratio

$$\frac{\text{number of students taking at least one fall and one spring test}}{\text{number of students taking at least one fall test}}$$

In the thirteen schools in this study, this ratio had the following values: .97, .94, .94, .93, .93, .90, .89, .86, .86, .75, .75, .67, .40. The three schools with lowest holding power are those in which credit can be obtained for 1 semester of geometry. Surely being in a school in which only 40% of students present for the fall testing were present in the spring is quite different from being in a school where almost everyone remains in geometry.

For the entire sample, 2491 students took at least one of the fall tests and 2065 of those took at least one of the spring tests, for a composite holding power of .83. In the ten schools with full year credit the holding power was 88%. Though a few students might have been absent for all two or three days of spring testing, it seems safe to say that in our sample about 1 in 7 students enrolled in geometry at the beginning of the school year were not enrolled at the end.

In Table 2 is given the year in school and age distribution for the ALL4 subsample. By comparing these with the corresponding distributions for the total sample, one sees that the students who did not take all of the tests in this study tended to be older.

Now we turn to test reliability. The reliability measures given here are calculated from the responses of all students who took the respective tests using formulas given in Guilford (1969).

The 19-item EG test for the CDASSG sample has Kuder-Richardson formula 20 (= Cronbach's α in this situation) reliability .77; Horst's modification gives .79 reliability, corresponding to a .89 reliability for a similar test of 40 items, higher than the .85 reliability on the 40-item CAP (compared to its published reliability of .89).

The van Hiele test, for purposes of reliability, is considered as five 5-item tests. The K-R formula 20 reliabilities (Horst modification reliability in parentheses) for the five parts in fall are .31 (.36), .44 (.48), .49 (.60), .13 (.13), and .10 (.11), and in spring are .39 (.43), .55 (.59), .56 (.59), .30 (.31), and .26 (.27). One reason for the low reliabilities is the small number of items; similar tests at each level 25 items long would have reliabilities .74, .82, .88, .43, and .38 in fall; .79, .88, .88, .69, and .65 in spring. The low reliabilities at levels 4 and 5 may be a byproduct of the lack of specification of the van Hiele theory at these levels.

For the proof tests, each form and each variable must be considered. The values of Cronbach's α for the forms, respectively, are .86, .85, and .88 for PrfTOT and .79, .81, and .86 for PrfCOR (Senk, 1982).

CHAPTER VI: RESULTS REGARDING VAN HIELE LEVELS

The results are organized here by the questions stated on pages 1 and 2 of this report.

Question 1. How are entering geometry students distributed with respect to the levels in the van Hiele scheme?

The classical theory. Table 3 shows the distribution into van Hiele levels for all students who took the VHF test (the VHF subsample). and for those students who took the VHF, VHS, EG, and CAP tests (the ALL4 subsample). Because there were two criteria used for classifying students into levels, percentages are given for each criterion.

Three aspects of these tables are worthy of special note. First, despite the VH test being a rather crude device for classifying students, 70% of students were classifiable into a level on the 3 of 5 criterion and 88% on the stricter criterion. Given that one needs to reach the criterion at all levels from 1 to n and at no other levels so as to be classified at level n, and given that the 3 of 5 criterion can be satisfied at any level by 6% of the students just by chance, the differences in percentages can be attributed almost entirely to Type I error. Thus a high majority of our students were rather easy to classify into van Hiele levels.

Second, the VHF and ALL4 subsamples distribute themselves in about the same percentages, though with a slight tendency for the ALL4 subsample students to be at higher levels. Yet the ALL4 subsample, because it requires attendance on four days rather than one, and presence in the spring as well as the fall, would seem in theory to contain better students. A chi-square test comparing the ALL4 and VHF-with-ALL4-removed subsamples confirms that the distribution of frequencies among classical van Hiele levels is significantly different for both the 3 of 5 criterion ($p < .01$) and the 4 of 5 criterion ($p < .001$), even though corresponding percentages differ by no more than 3%. (For the 4 of 5 criterion, levels 3, 4, and 5 were collapsed into one category.)

Third, the choice of criterion markedly affects the van Hiele level assigned to a student. On the 3 of 5 criterion, only 9% of those classifiable are at level 0. The 4 of 5 criterion is tougher to satisfy and fully 34% of those classifiable are at level 0. The percentages of those classifiable who fall into level 1 are nearly the same, but about twice as many of those classifiable fall into level 2 on the 3 of 5 criterion as fall into level 2 on the 4 of 5 criterion.

The top half of Table 4 is a crosstabulation matrix of student classical van Hiele levels under the 3 of 5 and 4 of 5 criteria. The main diagonal (NW to SE) of the matrix counts those students whose van Hiele levels are the same under the two criteria. Of the 1046 students who are assigned classical van Hiele levels under both criteria, only 533 (51%) have the same level under both. Thus van Hiele level is not nearly as fixed as would be suggested by the theory.

That it is easy to classify a student into a level is a plus for the van Hiele theory; that the student may have different levels dependent only upon the choice of criterion for reaching is a minus. The former suggests that a better test might classify even more students; the latter suggests that the levels are merely standards of competence characterized by increasing difficulty.

The modified theory. Recall that there is question regarding level 5 of the theory. Table 5 gives the distributions of students corresponding to those in Table 2, but according to the modified van Hiele level designation, i.e., with level 5 omitted. The percentage of students that now can be assigned a level increases markedly in every subsample, and the lowest percentage is now 85% rather than 70%. Specifically, about half the students not assignable under the 3 of 5 criterion and about a third of those not assignable under the 4 of 5 criterion can be assigned if level 5 is removed. What happened is this: Some level 5 items turned out to be easier for students than items at lower levels. So, e.g., some students would satisfy the criterion at levels 1, 2, and 5 and not be classifiable under the classical theory, but classified into level 2 under a modified theory. Thus, level 5 continues to be a problem for the theory, and deleting level 5 gives a better fit than before. Indeed, being able to classify 85% to 92% of the population far exceeded our expectations, given the probability of Type I error.

The bottom half of Table 4 shows that only 52% of students (658 of 1268) are assigned the same modified van Hiele level under the two criteria. This is consistent with results found for the classical theory.

If the theory is assumed, a student should have only one level. Which level is then the correct one? We do not have an answer, but a number of avenues for resolving the difficulty are possible.

Increasing the number of items at each level would lessen the impact of guessing and of a response on just one item. At higher levels (levels 4 and 5, but possibly also level 3) the criterion could be made easier. For instance, an 80% criterion could be used for responses at levels 1 and 2, and a 60% criterion at levels 3, 4, and 5. With a greater number of questions, these percentages could be fine-tuned even more. Van Hiele himself discusses transitions from one level to the next and one could assign levels between levels; e.g., the transition from level 2 to level 3 would be characterized by reaching a high criterion on levels 1 and 2 and some middle criterion on level 3.

If we had only used the 4 of 5 classical criterion, the reader would have learned that we could assign a unique (!) van Hiele level to 88% of entering geometry students and might have assumed that there actually existed some clearcut evidence that this level was fixed. We could have discussed a student J.M. who is at level 2 as if a level 2 student had such a fixed profile that the reader would immediately conjure up the profile. By analyzing various assignment criteria, we have demonstrated how much one's van Hiele level depends upon criterion for the level, even when the questions are not changed.

That would seem to weaken the theory a great deal, but a theory is a construct, and despite its validity or lack thereof, it is only as good as it is useful. Using an analogy from a different area, there may not be such a thing as one's superego, but some who study personality find the superego construct to be helpful both in analyzing behavior and in treating patients. Van Hiele levels may only be scores on a test, changeable from test to test depending upon the difficulty of the items, but they still may be useful for analyzing behavior and treating students.

Forced van Hiele levels. Having modified the van Hiele level determination scheme only slightly by ignoring level 5, and being able to fit so many more students into the scheme, we wondered whether further modifications in some reasonable way might enable almost all students to be assigned a van Hiele level. Whereas the modified van Hiele levels take into account only a perceived weakness in the theory, what we call *forced van Hiele levels* have an additional guiding principle, that the fixed sequential nature of the levels is valid, so a student whose responses do not fit the sequence is probably demonstrating random fit rather than a weakness of the theory.

To determine a student's forced van Hiele assignment, the following procedure is used. First a criterion is chosen (3 of 5 or 4 of 5) and a student is assigned a modified van Hiele level according to that criterion. Then responses of those students who do not fit that modified van Hiele level are examined. A student is assigned to level n if (a) the student meets the criterion at levels n and $n-1$ but perhaps not at one of $n-2$ or $n-3$, or (b) the student meets the criterion at level n, all levels below n, but not at level $n+1$ yet also meets the criterion at one higher level. A schematic description of the 32 possible profiles of meeting or not meeting the criteria at the 5 levels together with the corresponding weighted sum and assignment of forced van Hiele levels is given in Table 6.

Table 6 also includes the numbers of students with each profile at either criterion. Thus from this table the reader can verify all of the counts of van Hiele levels in the previous tables for the VHF subsample. This table gives a dramatic picture of the high percentage of students who fit the van Hiele profiles suggested by the classical and modified theories. There are 11 profiles with more than 2% of the students in them at the 3 of 5 level, and they include 5 of the 6 profiles fitting the classical theory and 8 of 10 fitting the modified theory. The three most common profiles not fitting the modified theory (those corresponding to weighted sums of 5, 9, and 11) all can be assigned to a forced van Hiele level. In fact, all but 7 students can be assigned a forced van Hiele level at the 3 of 5 criterion, and all but 5 students fit a forced level at the 4 of 5 criterion. This constitutes 99.7% of those who took the test.

Forcing a van Hiele level is tantamount to assuming that the theory does hold and that those students who do not fit would have fit if there had been more items or better items to minimize random error misclassification. This would be assuming what the project is attempting to test, so forced van Hiele levels are not used in any subsequent analysis. But the analysis with forced levels substantiates the existence of reasonable procedures under which almost every student can be assigned a level.

Wirszup's claim. Wirszup (1976) has claimed that "The majority of our high school students are at the first level of development in geometry, while the course they take demands the fourth level of thought." We can test the first half of this claim even with the differences among the criteria.

Table 7 gives the percentages of entering geometry students at level 0 or 1. The percentages are affected by the base one selects (all geometry students, or just those classifiable) and by the criterion, and range from 35% to 81%. We believe that these data show Wirszup's claim to be correct, if not to the letter, at least in spirit. What is indeed even more depressing are the percentages of students who do not reach level 1.

However, this test of Wirszup's claim only covers the first days of the year. Possibly students begin at low levels but very quickly, by the time they must do proof, are at higher levels. Or it is possible that the level a student is at does not affect the student's later ability to succeed in geometry. The next question concerns one of these issues, the changes in student van Hiele levels from fall to spring.

Question 2. What changes in van Hiele levels take place after a year's study of geometry?

In answering this question, the largest subsample that can be used is the intersection of the VHF and VHS subsamples. However, we utilize the smaller ALL4 subsample (which includes only students who also took both the EG and CAP tests) to enable comparison with results of analyses relating van Hiele levels to scores on content tests. There is a difference between the VHS levels of the ALL4 and the (VHF $\cap$ VHS - ALL4) subsamples (for the 3 of 5 criterion, a chi-square test shows significance at the .01 level), indicating that those who were absent on one of the days of the content tests are different from those who were present on all days. This is to be expected; better students have better attendance records on the average. However, since the ALL4 subsample numbers 1596 and the VHF $\cap$ VHS subsample 1807, the absentees have only small effects on the overall percentages. So the reader should note that percentages given here reflect a slightly better student population than would be found if we had made an effort to catch up with absentees.

Changes from fall to spring. The four sections of Table 8 exhibit crosstabulations of fall and spring van Hiele levels for the ALL4 subsample on each of the four assignment criteria. For each fall level, i.e., for each row in each table, the median spring level has been underlined. There are several patterns involving these medians. Classical and modified van Hiele levels behave similarly throughout. With the 4 of 5 criterion, the students at any level k in the fall have a median spring level k+1. (The three students at fall level 4 are too small in number to constitute meaningful input into this pattern.) That pattern holds for the 3 of 5 criterion with two exceptions: fall students at level 0 have median spring level 2 and those at the highest possible levels in fall remain there in spring (having no higher level to reach).

It is incorrect to summarize this pattern by asserting that students above level 0 in the fall tend to go up one level by the end of the year. On the 3 of 5 criterion there are a few cases where the median level does not represent the plurality and only for those who have modified fall level 3 does the median level in the spring represent a majority of students. That there is great variability is evident from Table 8 but exhibited in slightly different fashion in Table 9, where the data are organized not by assignment criterion but by fall level. There we see that for the fall levels in which students most commonly appear, those who fit in the spring very roughly split into thirds: a third go up one level; a third exhibit "great growth", increasing two or more levels; the final third exhibits "no growth", staying the same or decreasing their level.

Regardless of the relationship of the VH test to any sort of conceptual levels, one would expect students to perform better in the spring than in the fall on a test identical to that given to them in the fall. From 4% to 6% of the ALL4 subsample, constituting 7% to 8% of those who fit in both fall and spring, had a lower VHS level than VHF level. These percentages could constitute evidence of a lack of reliability of the test, random response error, or a violation of that part of the level theory which would seem to preclude regression in levels.

Comparison with Dina van Hiele's results. Dina van Hiele (1957) reports having been able to lead students from level 1 to 3 in 70 lessons, 20 lessons to go from level 1 to level 2 and 50 more lessons to go from level 2 to level 3. If either the classical or modified criterion is used as a basis for measuring level, Table 9 shows that 20% to 36% of students who begin the year at level 1 increase two or more levels, what Dina reports her students were able to do. However, a typical year allows for about 140 teaching lessons between our times of testing, twice as many as Dina used, and students in our sample average in age 4 years older than Dina's students. So it would seem that teaching the levels can accelerate children through them, though we do not have data from Dina regarding percentages of students in her class who did not increase two levels, nor data concerning students who may have started at a level higher than level 1.

Spring van Hiele levels. After the changes that have occurred during the year, it is appropriate to look at the distribution of van Hiele levels in the spring. Table 10 does that for the classical and modified van Hiele levels. (The reader may wish to compare Table 10 with the corresponding Tables 3 and 5 for the fall levels.) Two subsamples are given, the larger VHS subsample consisting of all who took the van Hiele test in spring, and the ALL4 subset of VHS. The percentages are within 2% but a chi-square test shows those in VHS and not in the ALL4 to differ significantly (at the .001 level) on all four criteria. Again the ALL4 subsample is at slightly higher van Hiele levels than the larger subsample of all those who took the test.

The results are generally rather depressing. On the easier 3 of 5 criterion, about 40% of the students who fit in the spring are at levels 0, 1, or 2. If the harder 4 of 5 criterion is used, over 70% of students are at these levels. The kinds of questions at these levels are those many geometry teachers would hope students would know from junior high school mathematics classes, and thus we may conclude that about half of all geometry students leave senior high school geometry with only a junior high school conception of the subject.

There is a sizable group (from 13% to over 40% of the VHS subsample, depending upon the criterion and base used for calculation) that finish the year at levels 0 or 1. These students cannot accurately identify properties of figures that were drawn in front of them, despite a year's work in geometry. For these students it would seem that the study of geometry is either inappropriate or has been accomplished in an ineffective manner.

Few students (barely a quarter of the population at most) are at levels 4 or 5, the levels at which, according to the van Hiele theory, students are able to understand proof. But perhaps being at level 4 is not a requirement for being able to write proofs or otherwise perform well in geometry, and perhaps being at lower levels does not affect geometry achievement. Question 3 (page 43) addresses this question.

Fit vs. no fit. In each of Tables 7 through 10 a sizable minority of students is classified as "nofit", meaning that each of these students has satisfied the indicated criterion at some level n but not at all levels below n. Having found differences between those who are in the ALL4 subsample and those who are not, we wondered whether there were differences between those who fit and those who did not.

Specifically, we asked whether those who fit one of the four level schemes (classical or modified, 3 of 5 or 4 of 5) in the fall were more likely to fit in the spring than those who did not fit in the fall. For the classical levels, a 2 x 2 chi-square analysis gives a negative answer with $p < .01$; those who do not fit in the fall are more likely not to fit in the spring. For the modified levels, the same type of analysis gives a positive answer; those who do not fit modified levels in the fall are just as likely to fit in the spring as those who do fit in the fall.

The reader who is interested in doing further analyses may wish to examine Tables 11 and 12, in which are found crosstabulations of the weighted sum scores for the van Hiele tests in fall and spring for the ALL4 subsample. The designations Cn and Mn by the scores (n ranges from 0 to 5) mean that the sum score next to them puts a student on classical level n (Cn) or modified level n (Mn). For example, a score of 31 puts a student at classical level 5 (C5) and modified level 4 (M4). The tables show with striking clarity how many of those who do not fit the classical levels in fall or spring on either criterion are picked up by the corresponding modified levels.

Fitting is not just a nice thing to have for purposes of later analysis; for the van Hiele theory fitting is a verification of a student going through the levels in order. That modified van Hiele levels fit more students more consistently than the classical van Hiele levels is further evidence that level 5 is out of kilter with the other levels.

Question 3. To what extent are van Hiele levels related to concurrent geometry achievement?

Here achievement is measured by performance on the EG, CAP, and proof tests. Question 3 asks for an analysis of the results on the fall van Hiele test relative to the EG and of the spring van Hiele test relative to the CAP and proof tests. Specifically, we look either at relationships between distributions on these tests or at differences in achievement between students at adjacent van Hiele levels. These analyses involve subsamples not used before.

ALL4:C3fit = students in ALL4 with a vH level in fall and spring using the classical 3 of 5 criterion

ALL4:C4fit = students in ALL4 with a vH level in fall and spring using the classical 4 of 5 criterion

ALL4:M3fit = students in ALL4 with a vH level in fall and spring using the modified 3 of 5 criterion

ALL4:M4fit = students in ALL4 with a vH level in fall and spring using the modified 4 of 5 criterion

Since not all students took the proof tests, each analysis involving the proof tests makes use of an _ALL5_ subsample, consisting of those students who took all of the EG, VHF, VHS, CAP, and proof tests. When van Hiele levels are being compared to results on the proof tests, subsamples corresponding to those named above (ALL5:C3fit, etc.) are used. Because there are three forms of the proof tests, every analysis involving the proof tests is done three times, once for each form, and involves roughly one-third of the corresponding sample.

While all of this may seem very complicated, each subsample is consistent in being the largest subset of ALL4 that is appropriate to the particular analysis. Table 13 gives the number of students in each of these subsets.

Correlations. Correlations done with van Hiele levels suffer two fundamental weaknesses. First, they require considering the van Hiele levels as evenly spaced, i.e., for any n the distance between levels n and n+1 is constant. This is not one of those properties claimed or suggested for the van Hiele theory and, indeed, this property is theorized not to be the case if one extrapolates from the amounts of time it took Dina to lead her students from level 1 to level 2 (20 lessons) and from level 2 to level 3 (50 lessons). Second, as there are only five levels, there are only six values (the sixth value being the nonattainment of the lowest level) that the VHF or VHS variable can assume. (We assign no value to those students who do not fit a particular criterion.) With only six values, the VHF and VHS variables are not "fine-tuned" in the sense of being sensitive to subtle changes in student geometry development. Thus one would expect smaller correlation coefficients when the van Hiele variables are involved than with tests having a greater number of possible scores (see Guilford, pp. 352-3).

Despite these weaknesses, correlations between van Hiele level and concurrent knowledge of geometry are uniformly high. Entering geometry knowledge correlates between .58 and .61 with fall van Hiele level (Table 14).* For the ALL5 subsamples (Table 15), the range is .52 to .64 with a generally slightly lower average than for the four ALL4 subsamples. The wider range is due to the greater number of coefficients calculated for the ALL5 subsamples (12 vs. 4) and the mutual exclusivity of the forms. In all cases the correlation coefficients are statistically significant at the .0001 level.

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*The closeness of the correlation coefficients for the four ALL4 subsamples is to be expected, because the subsamples overlap. The values are independent only for the various forms of the proof test, where the samples do not overlap.

Spring van Hiele levels correlate even higher with CAP than fall van Hiele levels with EG, the coefficients ranging from .61 to .67 for the various ALL4 subsamples and .56 to .69 for the ALL5 subsamples. These higher values may be due merely to the statistical properties resulting from the CAP test having a larger number of cells than the EG, 41 vs. 20.

Turning to correlations between VHS and proof performance, recall that two measures of proof-writing performance were calculated. PrfTOT is the standard total score on the six items, with a maximum possible value of 24. PrfCOR is the number of full proof items on which the student scored 3 or 4 points. PrfCOR ranges from 0 to 4, since there were four proof items on each form of the Proof test. Correlations between VHS and PrfTOT range from .57 to .69, and between VHS and PrfCOR they range from .51 to .68. These are nearly the same as the correlations between VHS and CAP.

The closeness of the values of these correlations is shown by looking at the averages of these correlation coefficients.

VHF vs. EG	.59 (ALL5)	.59 (ALL4)
VHS vs. CAP	.62	.65
VHS vs. PrfTOT	.62	
VHS vs. PrfCOR	.60	

The closeness suggests that van Hiele level is no more related to the ability to do proof than to the ability to answer standard multiple-choice questions. Yet one might expect that scores on two multiple-choice exams would tend to be more highly correlated than scores on one multiple-choice exam and one exam with open-ended questions such as the Proof test. Thus the relationship between van Hiele level and proof performance may be stronger than that between van Hiele level and standard content but the correlation coefficients may not show this due to the nature of the tests used in this study.

Even such high correlations as these do not imply or even suggest causality. It is possible that, to perform well on geometry content tests, one must be at a high van Hiele level. For example, perhaps despite our beliefs there are many questions on the CAP that require operating at higher van Hiele levels. Yet the converse is just as tenable, that to be at higher van Hiele levels one must have more knowledge of standard geometry topics. And there is the further possibility that the van Hiele level and geometry content tests are measuring approximately the same things. This last possibility may be the most reasonable of the three, since our van Hiele test items were based upon those quotes from the van Hieles that could be opertationally tied to language with which the students were likely to be familiar. Regardless of cause, one conclusion is clear: There is a strong relationship between performance on geometry tests and van Hiele level.

Of the four assignment criteria, the classical 3 of 5 (C3) generally yields the highest correlations. We explain this as follows: C3 has more cells than M3 and this overcomes the probable better theoretical accuracy of M3. C4 does not spread out students enough and M4 has all of the difficulties of C4 and M3. So C3, despite its weaknesses, correlates best.

The reader should note the very high correlations between CAP and either PrfTOT or PrfCOR (Table 15). The range, from .68 to .76, and the average .73 are substantially higher than the corresponding correlations between VHS and the Proof variables. Thus concurrent geometry knowledge is a better predictor of proof performance than van Hiele level, if one goes by correlations.

Differences between adjacent levels. Having found such strong overall relationships between van Hiele levels and concurrent knowledge, it is natural to ask whether students at adjacent van Hiele levels perform significantly different from each other. Table 16 gives the results of t tests of equality of means of EG scores for students at adjacent van Hiele levels. (In this and all subsequent analyses of this type, these t tests follow a one-way analysis of variance with VH level as the independent variable and geometry knowledge as the dependent variable which, as the earlier correlations suggest, shows the VH level to be a significant contributor to the variance at the .0001 level.)

On all assignment criteria, for $n \leq 3$ students at VHF level n score significantly higher on the EG than students at level $n - 1$ at the .0001 level. On the C3 and M3 criteria, level 4 acts differently from level 3 but not at that high a significance level. Thus differences between the lower van Hiele levels in fall consistently signal differences in entering geometry knowledge. On the C4 and M4 criteria, levels 3 and 4 are similar in EG performance, and on the C3 criterion levels 4 and 5 are similar in EG performance. So the highest two van Hiele levels (except for M3) do not signal any differences in entering geometry knowledge.

The same tests, conducted with CAP and VHS, show almost a reverse pattern (Table 17). In the spring, students at van Hiele levels 0 and 1 on the C3 and M3 criteria have nearly the same geometry knowledge, and students at all other levels have different knowledge except levels 4 and 5 on the C4 criteria. These discrepancies are easily explained. A VHS level of either 0 or 1 is so low that the student at either level is unable to grasp the kinds of geometric concepts needed to perform well on the CAP. A VHS level of 4 is sufficient to grasp the concepts required by the CAP, so there is no extra advantage to being at level 5. An alternate explanation is that the similar performance by those at levels 4 and 5 is further evidence that the existence of level 5 in the theory should be questioned.

We interpret Tables 16 and 17 as demonstrating the ability of van Hiele levels to signal concurrent performance on a typical test of standard content.

Means and t test analyses for the Proof variables are given in Table 18. The signals are similar but not as strong with respect to proof performance, perhaps because the Proof test is shorter and has three nonequivalent forms. Also, the effects upon PrfTOT are greater than the effects upon PrfCOR, perhaps because PrfTOT allows for greater variation in student scores.

Students at levels 0 and 1 in the spring tend to act alike on all forms for both PrfTOT and PrfCOR. Levels 1 and 2 tend to signal different performance on PrfTOT but not always on PrfCOR. The difference between the mean proof scores of students at levels 2 and 3 is significant on both measures, all forms, and all criteria. Students at level 4 score higher, but not always significantly higher, than those at level 3. Students at level 5 on the C3 criterion score significantly higher than those at level 4. In sum, though examination of the means shows that being at a higher VHS level tends to yield better proof performance (something the correlations showed as well), only between levels 2 and 3 can we say with authority that there is a substantial difference in performance.

The leftmost column of Table 18 includes the percentages of variance in the PrfTOT and PrfCOR distributions accounted for by the VHS level. These values, squares of the corresponding correlation coefficients, vary from .34 to .49 on PrfTOT and .27 to .47 on PrfCOR, with the contribution to PrfTOT never lower than the contribution to PrfCOR. Of the four assignment criteria, C3 contributes to PrfTOT the most. In comparison, CAP scores alone contribute between .46 and .58 of the variance in the distributions of the proof variables.

Effects of level on proof performance accounting for knowledge. Because CAP is more higly correlated with the Proof variables than the various van Hiele level distributions, it is natural to wonder whether VHS level adds anything after the effects of CAP are taken into consideration. For this reason ANCOVA analyses were undertaken with VHS level (with each of the four assignment criteria) as the independent variable, PrfTOT and PrfCOR as dependent variables, and CAP as the covariate. These show that VHS contributes significantly to the variance at the .0001 level even after CAP is accounted for.

Together, VHS and CAP account for between .51 and .63 of the variance on PrfTOT and for between .51 and .61 of the variance on PrfCOR (Table 19). But when CAP is brought into the picture, the differences between adjacent adjusted means on the Proof variables are not as great and the probabilities that there are no differences between adjacent means are greater. Only between levels 2 and 3 are the PrfCOR and PrfTOT means significantly different most of the time. On other adjacent levels there are always occasions when the adjusted mean at level n is lower than the adjusted mean at level n - 1. Thus VHS has a significant affect on proof performance even after geometry knowledge is taken into account, but a difference of one level indicates little unless that difference is between levels 2 and 3.

Wirszup's claim. Wirszup's comment that "the course they [high school geometry students] take demands the *fourth* level of thought" (1976, p. 96, emphasis his) refers to proof aspects of the geometry course. This claim underlies arguments for changing the geometry course to be more appropriate to the learning capabilities of the students (e.g., see Hoffer (1981), p. 14), and begs for analysis of relationships between spring van Hiele level and success on proofs.

PrfCOR has only 5 possible values, enabling display of cross-tabulations of VHS level and PrfCOR, as is done in Table 20. The no-fits have been included in these crosstabulations to display all of the scores and make possible more accurate analyses of the difficulties of these forms.

Which form is easiest or most difficult depends upon the procedure for judging difficulty. Based upon the means, Form 2 is easiest and Form 1 is most difficult. Based upon the ability to get at least one proof correct, Form 2 is easiest and Form 3 is most difficult. Based upon the ability to get all four proofs correct, Form 3 is easiest and Form 1 is most difficult. (This is a clear instance of the potential loss of information that can occur if one examines means alone.) These inconsistencies result from Form 1 having the hardest single proof item and the easiest proof on Form 3 being quite a bit more difficult than the easiest proofs on Forms 1 and 2. (See Appendix C for an item analysis of the Proof tests.) Despite these inconsistencies, the three forms possess some similar properties.

The rightmost column in Table 20 exhibits a very strong relationship between a student's spring van Hiele level and the probability that the student will be successful with proof. Here we see that the strictness of criterion (3 of 5 = weaker; 4 of 5 = stronger) determines the van Hiele levels that indicate success and that classical and modified criteria operate quite similarly. Specifically:

a student is quite likely ($p > \frac{2}{3}$) to experience failure in proof	iff	weaker VHS < 3 or stricter VHS < 2
a student has about an even chance of success or failure in proof ($.4 < p < .6$)	iff	weaker VHS = 3 or stricter VHS = 2
a student is quite likely ($p > .7$) to experience success in proof	iff	weaker VHS > 3 or stricter VHS > 2
a student will almost surely ($p > .87$) experience success in proof	iff	weaker VHS = 5 or stricter VHS > 3

Thus a van Hiele level of 3 on the weaker criterion or 2 on the stricter criterion acts as a guidepost. Above these levels, success in proof is likely, but below these levels failure is just as likely. Being at level 4 on any of the four assignment criteria promises a good chance for being successful at proof, but the stricter criterion is difficult enough so that level 3 with that criterion offers a reasonable chance ($p > .75$) of success as well. The percentages are remarkably consistent among the forms, leading one to say with reasonable confidence that Wirszup and Hoffer are correct when it comes to proof: Van Hiele levels are a very good indicator when it comes to predicting success on proof. Whether, however, one needs to be at level 3 or 4 in order to have a good chance of success depends upon the assignment criterion.

The oversimplification of assigning students to one of five levels may be manifested here. According to the van Hiele theory, there are transitions between the levels and it is quite likely that many students, having had much exposure to proof during a year's worth of geometry, would be between levels 3 and 4. It is possible that the choice of criterion (stronger or weaker) has the effect of assigning students who are between to the lower (if the stronger criterion is used) or the higher (if the weaker criterion is used) of the two levels. If that is the case, then these results, in which the weaker criterion at level n acts like the stronger criterion at level n - 1, are quite consistent with one another. A way of resolving this kind of situation for a student would be to assign to each student the mean of the van Hiele levels as calculated using the two criteria. We did not do any such calculations in this study.

Question 4. To what extent do van Hiele levels predict achievement after a year's study?

The same kinds of analyses used in answering Question 3 were used for Question 4, since predicting achievement eight months ahead (from September to May) poses the same statistical problems as relating achievement on tests given two days apart. As one would expect, however, the eight month interval allows any intervening variables more time to have influence and so we find the degree of relationship between fall van Hiele levels and spring achievement is generally not as high as between spring van Hiele levels and spring achievement. From the learning theory perspective, these intervening variables cloud effects of van Hiele levels enough to make Question 4 a weak sibling to Question 3. That is, the effects of van Hiele levels on learning are best tested by looking at learning simultaneously with examination of van Hiele levels. However, from the practical school-based perspective, in which decisions regarding placement of a student are made before the beginning of the school year and seldom changed due to performance except in the case of failure, one seeks guidance regarding the potential for the use of van Hiele levels to decide upon such placement. Nor is this use outside the theory, for if indeed the course is taught 2 levels higher than where the student is at, the theory asserts that failure is to be expected, for the teacher and students will not be able to understand each other. Thus we examined relationships between VHF level and performance on the CAP and Prf tests.

Correlations. The correlations between VHF and spring geometry knowledge as measured by the CAP range from .51 to .52 for the ALL4 subsamples (Table 14) and hover around these values for the ALL5 subsamples as well (Table 15). These correlations are .10 to .15 lower than the corresponding correlations between VHS and CAP for the ALL4 subsample, and average .10 (range -.01 to .20) lower for the ALL5 subsample. Still they are all significant at the .0001 level. Fall van Hiele level is a significant predictor of spring geometry knowledge as measured by a standardized test.

The correlations between VHF and CAP are also consistently lower (by .13 to .15 for ALL4 subsamples and by an average of .12 for the ALL5 subsamples) than the correlations between EG and CAP. So entering geometry knowledge is a better predictor of spring geometry knowledge than entering van Hiele level. However, for these subsamples, spring van Hiele level is a little better at predicting spring knowledge than entering van Hiele level. So, if a person wishes to predict scores on a standardized geometry test, look first at VHS, then at EG, and last at VHF.

Though there is a tendency for the correlations between VHF and either of the proof variables PrfTOT and PrfCOR to be slightly lower than those between VHF and CAP, the differences are minimal. This agrees with the corresponding correlations involving VHS and again suggests that van Hiele level as measured in this study is no more related to the ability to do proof than to the ability to answer standard multiple-choice questions. (The reader may wish to refer to the corresponding discussion of correlations after Question 3 for caveats regarding these conclusions.) Of the four assignment criteria, the C3 again generally yields the highest correlations.

Effects of level on CAP. Mean CAP scores were calculated for all students at each fall van Hiele level (Table 21). Except for levels 4 and 5 on criterion C3 and levels 3 and 4 on the stricter 4 of 5 criterion, every successively higher fall VH level makes a difference in CAP performance. Thus differences in fall van Hiele level not only signal differences in entering knowledge (as shown in Table 17) but also differences in ability to learn standard geometry content.

Since EG scores correlate higher with CAP than do van Hiele levels, we consider whether knowing the van Hiele level adds anything after the EG score has been taken into account. From an analysis of CAP scores, adjusted via ANCOVA for EG scores, together EG and VHF contibute .44 to .47 of the variance (compared with .26 to .28 for VHF alone) and .34 to .37 for EG alone, so that VHF adds about .10 to the amount of variance accounted for by EG (Table 22). This is a significant addition at the .0001 level. (This significance level is maintained by the van Hiele levels in all of the ANCOVA analyses we conducted.)

CAP means adjusted for EG for students at neighboring van Hiele levels are significantly different only about half the time, and only between levels 2 and 3 is the significance consistent. Thus one should not use VHF alone to predict later geometry performance on a standardized test, except between these two levels.

We have purposely not mentioned whether we consider .05, .01, .001, or .0001 as a key significance level because the choice is in some sense arbitrary and we prefer simply to give the probabilities and let the reader choose, but given the number of analyses done even in a single table, .01 is the highest probability we would consider significant.

For the proof tests, we are content with relaxed levels of significance if there are consistent results for all three forms.

Effects of level on Proof. Tables 23 and 24 give unadjusted and adjusted (for EG) means on the proof variables PrfTOT and PrfCOR for students at each fall van Hiele level. They correspond to Tables 18 and 19 which give the corresponding statistics for spring van Heile level.

We consider unadjusted means first. The effects of VHF upon PrfTOT and PrfCOR are nearly identical and rather consistent. Differences in van Hiele levels below 4 have a significant affect upon later performance, the performance of students who enter at levels 4 or 5 is not must difference from those who enter at level 3.

When the means are adjusted for entering geometry knowledge, many of the differences disappear. Only levels 2 and 3 on the M3 criterion and levels 0 and 1 on the M4 criterion show significant differences in mean proof performance on all three forms of the Proof test.

Taken together, the two variables EG and VHF account for less than .375 of the variance in Form 1 and Form 3 proof scores and less than .50 of the variance in Form 2. Thus there are quite significant intervening variables.

Despite these relatively low contributions to the variance, analysis of the crosstabulations of VHF and PrfCOR is fruitful (Table 25). Specifically, entering at lower van Hiele levels decreases the chance that a student will be successful at proof. If PrfCOR $\geq$ 2 indicates success at proof, then:

a student is likely ($p > .65$) not to experience success in proof	iff	weaker VHF $\leq$ 1 or stricter VHF = 0
a student has about an even chance of success or nonsuccess in proof ($.38 < p < .60$)	iff	weaker VHF = 2 or stricter VHF = 1
a student is quite likely ($p > .75$) to experience success at proof	iff	weaker VHF $\geq$ 3 or stricter VHF $\geq$ 2

Thus a van Hiele level of 2 on the weaker criterion or 1 on the stricter criterion acts as a guidepost in fall, just as the next higher levels do in spring. Below those levels, success in proof is unlikely; above the levels success is quite likely.

If PrfCOR = 0 is taken as denoting failure at proof (it hardly could be considered otherwise), then even on the weaker C3 criterion and only considering those students who stay with the course until its end, a student who enters geometry at van Hiele levels 0 or 1 has an almost even chance of failure at proof. What is particularly discouraging about this statistic is that fully 38% of the CDASSG sample enters geometry at these van Hiele levels. Thus in our study almost 2/5 of the geometry students enter the course at such low levels that they have an even chance of total failure at proof. Furthermore, in many schools the best students are placed into an honors or accelerated class, so that in a regular level class about half the students enter the class with a 50-50 chance of failure.

Yet further, even a student who enters at van Hiele C3 level 2 has a 20% chance of failure, a probability that psychologists (but not many mathematics teachers?) might consider quite high going into a course. Only 17% of those students who fit the levels on the C3 criterion were above level 2, so perhaps 5 of 6 students who enter geometry have some reason to fear failure when it comes to proof.

There are many ways to interpret these statistics. Here is one. Many students wisely opt not to take geometry because the odds are they will not succeed. A majority of students know very little coming into the course and will have to work hard to avoid total failure at proof, since about half with their incoming van Hiele level will fail. Not many students enter the course at high enough levels to be relatively assured of not failing, and fewer yet enter the course at levels high enough to enable them to expect success. If student success or failure is the criterion by which the geometry course is to be judged, then Wirszup and Hoffer are without question correct when they assert that the geometry course as presently constituted is inappropriate for a very large number of the students who enroll in it.

Question 5. What generalizations can be made concerning the entering van Hiele level and geometry knowledge of students who are later found to be unsuccessful in their study of geometry?

This question is a converse of the previous question, which asked whether van Hiele levels predict achievement. Here we ask whether poor achievement predicts van Hiele levels. There was a hope, when this question was first formulated, that the subsample for this question would include both those students who demonstrated lack of success by poor performance on some test and those students who had already withdrawn failing from geometry before the spring test administration. We have remarked that teacher identification of reasons for students leaving their classes was quite unreliable, and thus the only students included here are those who were present for the spring test administration. The subsamples studied are defined arbitrarily as not reaching a certain standard on the CAP or Proof tests.

VHF and low CAP. A score of 14 on the CAP was picked as the cutoff below which a student was considered to be unsuccessful. The following properties led us to select this cutoff. (1) A student who scores below 14 has correctly answered fewer than 1/3 of the items on the multiple choice test and, indeed, has answered at most 5 items more than would be expected by random guessing. (2) The student with a score below 14 is at the 18th percentile or below according to the national norms (see Table 34). (3) Scores below 14 comprise the bottom quartile of this study's population.

Displayed in Table 26 are crosstabulations of CAP scores below 14 and the corresponding classical fall van Hiele levels for these students. (Table 17 suggests that the modified levels act like the classical levels in this regard, and this was verified by a similar analysis not presented here.) Unsuccessful students come almost entirely from lower van Hiele levels. Of the unsuccessful students, 74% have fall van Hiele levels 0 or 1 under the easier 3 of 5 criterion, whereas only 44% of the rest of the population have these low van Hiele levels. More striking, only 3% of unsuccessful students have fall van Hiele levels of 3, 4, or 5 under this criterion, whereas 24% of the remainder of the sample have these levels.

The 4 of 5 criterion is too tough to distinguish unsuccessful students. Although 94% of unsuccessful students have a fall van Hiele level of 0 or 1 under this criterion, 75% of the rest of this subsample also have these levels. That is, many students who are later not unsuccessful entered with these low van Hiele levels.

A few related pieces of data are interesting. In the study, 140 students were at VH fall level 0 on the 3 of 5 criterion. Of these, 46 were not present for the CAP test administration. This compares with 556 of 2221 others who were not present, not a particularly large difference in percentages (33% vs. 25%), though statistically significant. This suggests that dropouts are poorer but not always the poorest students. Some students at fall VH level 0 perform quite well on the CAP, one scoring 30, one scoring 29, and 12 scoring above the national norm mean of 19.

VHF and low Prf. Being unsuccessful at proof is defined by the simple criterion PrfCOR = 0. The crosstabulations of Table 23 show that students who are unsuccessful at proof are more likely to have low fall van Hiele levels, as would be expected from the correlations between VHF and PrfCOR. A more succinct picture of the van Hiele levels of unsuccessful proof students is given in Table 27 by collapsing all non-zero PrfCOR scores and combining the three forms of the proof test (which can be done here because the criterion is independent of form).

In this regard, classical and modified levels act very much alike. We give results for the classical levels. About 71% of students unsuccessful at proof have fall van Hiele levels of 0 or 1 on the easier 3 of 5 criterion, compared to 37% of those not unsuccessful. About 44% of unsuccessful students have fall van Hiele level 0 on the stricter 4 of 5 criterion, compared to 20% of those not unsuccessful. Thus students unsuccessful at proof are about twice as likely as the more sucessful others to have these low van Hiele levels. Clearly seen from this Table is the probability of about .5 that a student with VHF level 0 by any criterion will not be able to do a single proof by the end of the school year.

Question 6. To what extent is the geometry being taught to students appropriate to their van Hiele levels?

Results and remarks concerning this question should be considered as speculative, since data concerning the levels at which the geometry is being taught are indirect. There was no monitoring of classes during the year of the study. It is indeed possible that geometry classes are taught in quite appropriate ways to students, but the students are not motivated to do the work or are otherwise distracted and do not perform as they might. Thus while it is possible that a geometry course is inappropriate to a student, it is also possible that the student has behaved inappropriately in a course quite suitable for him or her.

In the discussion regarding the previous question, it was noted that the C3 criterion works well for purposes of identifying students in the fall who have a great probability of not being successful or of being successful. We restrict our analyses here to that criterion.

Students who enter at van Hiele level 0 have a probability of greater than .5 of being unsuccessful at proof, so should not be placed in a proof-oriented course. Similarly, students who enter at van Hiele level 1 have a probability of about .4 of not being successful at proof and probably should be counseled away from such courses. In Table 28 fall van Hiele levels are contrasted with track and rectangles have been drawn surrounding those students whose van Hiele levels indicate possible misplacement into a proof-oriented course. These rectangles include 31% of the students in our sample who took the VH test in the fall.

On the other hand, students who enter at van Hiele levels 3 or higher on the C3 criterion have a high probability of some success with proof ($p > .85$ that these students have $\mathrm{PrfCOR} > 1$) and so such students should be placed in a proof-oriented course. The numbers of students who have been misplaced under this criterion are indicated by the smooth oval in Table 28 . One might argue that an even chance of success at proof warrants placing a student in a proof-oriented class, and then the oval should be extended to include the area we have enclosed by the dashed oval. This larger area includes about 2.5% of the VHF subsample.

Putting this information together, according to the van Hiele test, about 34% of students are misplaced. If we use as the base of the percentages the number of students to whom a van Hiele level can be assigned (thus deleting the nofit students), the percentages increase to 44% placed in proof-oriented course incorrectly and 4% placed in non-proof course incorrectly, for a total misplacement of almost half of the students now taking secondary school geometry.

This may explain why a significant percentage (c. 25%) of first-year algebra students choose not to enroll in geometry. They may be making a very reasonable decision given their odds of success.

Those schools in which all students are placed into the same geometry class (i.e., there is no tracking) contribute heavily to these percentages. In untracked classes, 57% of students for whom there is a van Hiele level are at a level too low to have a high probability of success in a proof-oriented class. The corresponding percentage for students in schools with two tracks is 48%, and with three tracks is 27%.

Since both untracked and two-track schools in our sample do not have a non-proof option, the discrepancy in percentages (48% and 57%) must be considered due to other factors than tracking. This confirms the roughness of the percentages under discussion here. Still we may conclude that the offering of a non-proof alternative to the standard geometry course decreases the probability of misplacement by quite a significant amount, perhaps cutting the potential mismatches in half.

As any person acquainted with secondary school geometry teaching knows, there are degrees of emphasis upon proof, and the above discussion may do injustice to the sensitivity of geometry teachers by considering only two options for the geometry course: proof-oriented and non-proof-oriented. What we have concluded to be a misfit of van Hiele level and course emphasis is based upon identifying those fall van Hiele levels that are associated with non-success on proof; an alternate explanation is that teachers who believe their students are not ready for proof purposely cut down the amount of time devoted to proof, and reduce their expectations of proof competence accordingly. The result is that these students do not perform well on a proof test. We may think there has been a misfit of course and student when what has occurred is a mismatch of course and (proof) test.

As an argument in favor of the multiplicity of ways in which geometry courses adapt to students, we note that of the 13 schools in this study, only two had adopted the same textbooks for use in the same way; these two schools utilized the same book (Jurgensen et. al., Modern School Mathematics) for all of their students. While three other schools used this book with some students, they used different other books.

Table 29 provides a way of comparing van Hiele level and texbook. The three textbooks are the ones used by four or more of the thirteen schools in this study. (Eight other textbooks and teacher-made materials were used in these schools.) Only the use of the Moise and Downs Geometry, a highly sophisticated and proof-oriented geometry text, by School 9 strikes us as out of line. In this school, 59% of students with C3 van Hiele levels in fall are at levels 0 and 1. The other schools using the Moise and Downs text are tracked and use this text only in their top track.

Question 7. To what extent do geometry classes in different schools and socio-economic settings differ in the appropriateness of the content to the van Hiele level of the student?

This study cannot adequately answer this question, for the number of schools is too small to enable any generalizations to be made. As Table 1 indicates, only one size and type of community was represented by more than two schools. Consequently, whatever effects one might wish to attribute to socio-economic settings could also be attributed to other characteristics of the schools in the study (size, region of the country, tax base, percentage of students enrolled in geometry, etc.).

However, one important characteristic among our small sample of schools can be verified: they differ widely in both entering van Hiele levels (Table 30) and entering knowlege (Table 31). These differences may not be due at all to socio-economics; in the case of the school with the highest means they are due at least in part to the existence of an option for slower students in which a full year of geometry is not studied. The order of schools in Table 30 should be taken very lightly, for the reader can verify that the order of schools would be rather different if the mean fall van Hiele level on the stricter C4 criterion had been used. Furthermore, school mean scores on the entering geometry test are in substantially different order. Thus, though it appears that there might be some patterns in Table 30 regarding STOC and the ability of the incoming geometry student, these patterns disappear in Table 31 .

The differences in entering van Hiele levels is so great that the *mean spring* C3 van Hiele of five schools is lower than the *mean fall* van Hiele level of a sixth. One school's mean spring C3 van Hiele level is lower than the fall mean of all but two others. Thus we conclude that in some schools students know more geometry at the beginning of the school year than the students in other schools know after a year's study. This result agrees with similar results found in algebra (Swafford and Kepner, 1978).

One often sees discussed individual differences with respect to students; seldom are the principles applied to schools. Most of us fall into the habit of discussing "the" geometry curriculum or "geometry students" as if these terms applied equally well to all schools. The picture that emerges from Question 6 is that tracking enables schools to better tailor courses to the entering characteristics of geometry students. The picture that emerges from this question is that schools can be as different as tracks, and that a geometry course appropriate for most students in one school may be inappropriate for most students in another. Van Hiele levels may not be any better at judging appropriateness than a content test such as the EG, but they provide a second way of judging this appropriateness.

CHAPTER VII: OTHER RESULTS

In this chapter the entering geometry (EG), standardized test (CAP), van Hiele test (VH) and proof tests are examined individually, without reference to their possible connections with van Hiele levels, for the purpose of assessing the level of student performance. The results of these examinations are rather depressing. We also look at the presence or absence of differences between the sexes and among the schools in performance on these tests.

Performance on EG. Recall that the EG test (Appendix A) was adapted from a 50-item test used in the early 1970s in an Ohio State study of geometry knowledge among entering geometry students. Of the 19 items used in our study, item 20 has no counterpart on the OSU test and on items 10 and 13 the numbers were changed to lessen the possibility of students getting correct answers by an incorrect process. Consequently sixteen items can be compared (though on two of these wordings were changed and on one a single foil was changed). Table 32 gives percentages correct in the OSU study and in our study for these sixteen items.

The mean percentage correct for these sixteen items was 62% in the OSU study and 54% in the present study, indicating that the EG subsample performed 8% below the OSU sample. There is no way to determine whether the samples are comparable; it could easily be that the OSU sample consists of brighter students or of students in school systems that do particularly good work with geometry at the junior high school level. However, by subtracting from the means, the relative differences between items are easily compared. This indicates that the EG subsample performed particularly poorly on items dealing with angles and perpendicular and parallel lines. They performed better on content dealing with triangles.

On an absolute scale, the performance cannot be considered satisfactory. No item was correctly answered by 80% of the EG sample, even though some of the items involve only straightforward applications of terminology or formulas. For example, only slightly more than half of the students (52%) could calculate the area of a square given its sides. Only 72% could calculate the area of a rectangle given its length and width. Only 62% could identify a segment as a radius of a circle. And so on.

The implications of this quality of performance are clearer when the enrollment in high school geometry is taken into account. As noted in Chapter V, only slightly more than half of all seniors in high school have taken or are enrolled in high school geometry. With an optimistic assumption that those students who do not take high school geometry know as much geometry as those who do, countered by the pessimistic assumption that no geometry is learned outside of geometry classes, the conclusion is reached that a substantial percentage of adults (perhaps 15% to 25%) know not even the simplest geometry notions.

What seems to be the case is that junior high school teachers neglect to cover many aspects of geometry, thinking that their students will encounter geometry later. They do not realize that almost half of their students will never enroll in a formal geometry course.

Performance on VH test items. Appendix B contains the van Hiele test and item analysis of that test. While in previous chapters we have been concerned only with the attainment of particular van Hiele levels, here we analyze the performance on selected individual items.

In fall, 10% of students think a rectangle is a square (item 1) and 20% think other quadrilaterals might be a square (item 4). Students can identify rectangles at this time (item 3) but over two-thirds think a square is not a rectangle (item 13). What is discouraging is that these percentages do not change much from fall to spring; still two-fifths of students in spring think a square cannot be called a rectangle.

Work with triangles is no better. About one-third of students in fall think a long thin triangle is not a triangle (item 2) and do not know that isosceles triangles have two congruent angles (item 9). In the spring one-fifth of students still incorrectly respond to these trivial items.

No item dealing with reasoning to a conclusion (items 11-12, 14-18, 20-25) was correctly answered by more than half the students in the fall or two-thirds of the students in the spring. Only 28% of students can order simple propositions (item 17) and 44% think a statement implies its converse (item 18). The needs for undefined terms and assumed statements (postulates) are responded to almost randomly (item 19), even poorer results than found for this item in a 1968-69 study (Usiskin, 1969). The meaningsof mathematical impossibility (item 22), invention in a mathematical system (item 23), and definition (item 24) are foreign to about three-fifths of students. Thus the majority of students do not understand reasoning or operation in a mathematical system.

Performance on CAP. As a standardized test, the publisher gives national norms for performance on the CAP test. Unfortunately, there is no indication regarding either the sample size or the procedure for selecting schools in the norming process, so one cannot make many conclusions from comparing the present CAP subsample with the national norming subsample.

The percentages of students correctly answering each item on the CAP test are given in Table 33. In total, the mean score of 18.73 for the CAP subsample (n = 2015) is 2% below the norm mean of 19.65. This is reflected in the comparative percentile ranks (Table 34).

The greatest differences favoring the national norming sample are on items 17 and 25, dealing with coordinate geometry and inequalities, respectively. The greatest difference favoring the CAP subsample is on item 23, a numerical problem involving right triangles and the Pythagorean theorem. These are the three of forty items in which there was more than a 10% item difference between norming and CAP samples, having adjusted for the 2.2% total difference.

Generally the performance of both the norming and the CAP sample is low. It is hard to believe that, after a year of geometry, 18% to 20% of students cannot identify vertical angles, 26% to 30% cannot indicate what additional information would be needed to use the SAS theorem, 44% to 47% cannot find the perimeter of a square from its area, and 65% to 68% cannot calculate and subtract the areas of two circles to find the area of the space between them. If so little is learned, what is being taught?

Performance on Proof. Item analyses are given along with the proof items in Appendix C. PrfCOR and PrfTOT data for the entire Prf subsample (n = 1520) are given in Tables 35 and 36. Senk (1982) has reached the following conclusions from this data.

1. About 70% of students can do simple proofs requiring only one deduction beyond those made from the given.
2. Achievement is considerable lower on proofs requiring auxiliary lines or longer chains of reasoning.
3. Only about half the students can do any more than simple proofs.
4. Writing proofs is not an "all" or "nothing" task. Among the half of the population that can do more than simple proofs, there is a wide range of proof-writing achievement.

Senk's third conclusion is based upon the notion that on each form there was one very simple (some would say "trivial" proof). Because the forms are not equivalent, statistics that combine them are not meaningful for in-depth analysis, but totalling up the rows for each PrfCOR value in Table 35 yields the following percentages: 29% got no proofs correct, 21% got one proof correct (and thus 50% got at most one proof correct), 18% got two proofs correct, 18% got three proofs correct, and 13% got four proofs correct. The difficulty of the last proof on Form 1 serves to bring that last percentage down by 3 or 4 percent.

Defining a student to be successful at proof if the student has a PrfCOR score greater than or equal to 2, i.e., if the student can do half of the proofs correctly, one finds similarity even among these three forms with such different other properties, for the percentage of successful students ranges between 50% and 54% on the three forms. This leads to a possible explanation in Skinnerian terms for the continuing teaching of proof in high school geometry despite the many students who have difficulty. Extinction of a behavior is considered most difficult if the behavior is reinforced approximately half of the time. Here we can confirm that teaching proof leads to success in just about half of geometry students. This amount of success is enough to counter teacher frustration with the failures and so keeps the teaching of proof from being extinguished.

The PrfTOT distribution (Table 36) is, as one would expect, similar to that found for PrfCOR. Those students for whom PrfCOR = 0 are generally found with PrfTOT scores < 9. There is a significant percentage of students (about 16%) with PrfTOT < 5, indicating that many students who get no proofs correct are not generally able to make even one deduction from given information.

Combining this data with the participation data (Chapter V), the secondary school population roughly fits the following profile with respect to proof competence.

Of all United States high school students, approximately:

47% do not take geometry
6% take geometry but drop out before the end of the year
7% are enrolled in a non-proof geometry course
11% study proof but cannot do anything with it
9% can only do trivial proofs
7% have moderate success with proof
13% are successful with proof

We remind the reader that the sample for this study was not randomly selected and that these percentages must be regarded as rough (except for the first, which is off by at most 2%). Still, it seems that the geometry course, in its present form in which proof is the dominating force behind the scope and sequence, is reaching only approximately 30% of high school students, and a third of those reaching only in a marginal way.

Sex differences. Many of the remarks in this section are derived from the papers of Dees (1982) and Senk and Usiskin (1982). The former concentrates more on the EG, VH, and CAP tests; the latter concentrates exclusively on the Proof tests.

With regard to achievement on the EG test, the mean score for boys is one point higher than the mean score for girls (Table 37). This one point difference carries over to the CAP test at the end of the year, and both differences are statistically significant at the .001 level. They are the only consistent sex differences found in this study. When the CAP scores are adjusted using ANCOVA with the EG score as covariate, the sex differences disappear.

With regard to proof writing ability, boys do slightly better than girls on all three forms as measure by PrfTOT and in two of the three forms as measured by PrfCOR. However, when these scores are adjusted using ANCOVA with the EG score as covariate, the girls means are higher on all three forms on both PrfTOT and PrfCOR. One of these differences favoring the girls is significant at the .05 level, but this seems to be just a chance occurrence.

These results are consistent and lead us to two conclusions, neither of which is universally accepted by the research community or the public.

1. Regardless of the existence or nonexistence of differences in spatial ability between the sexes, these seem to have little if any bearing on potential differences in the ability to learn geometry, for that ability is equal between the sexes.

2. The ability to perform the reasoning processes involved in geometry proof, considered perhaps the most difficult of all higher order thinking processes found in mathematics, is equal between the sexes. Thus we dispute any assumption that boys are inherently better than girls at the kinds of reasonings involved in abstract mathematics or in problem solving.

While the second conclusion above may seem quite bold given the specific nature of the study, we feel justified in it because geometry proof is virtually unique among mathematical topics in that it is not learned outside of geometry classes even by the most talented or interested students. Thus, without having planned it, this study set up a controlled experiment in higher order mathematical thinking whereby the sexes were equally exposed to a concept both in class, where the treatment was obviously the same, and outside of class, where there is no study of geometry proof and no exposure in preceding years as well. That there were no plans to analyze our data by sex gives the study a double blind quality (neither the experimenters nor those in the study knew that sex was one of the variables that might be considered) which strengthens the validity of the conclusions.

Sex differences on van Hiele levels are difficult to explain. Frequencies of van Hiele level by sex are given in Table 38. Chi-square statistics were calculated for each of the eight tables, and indicate no sex differences in fall van Hiele levels, but consistent differences in spring van Hiele levels regardless of assignment criterion. An ANOVA, with sex as dependent variable and VHS as the continuous dependent variable (omitting those without levels) confirms what can be seen from Table 38, that the differences favor the males. The greater proportion of males at the higher spring van Hiele levels arises from sex differences in growth for those who begin at all lower van Hiele levels. These differences are difficult to explain because they occur despite the high correlations between VHS and the CAP upon which girls' scores are as close to boys as the VHF and EG were in fall, and despite the strong relationship between VHS and the proof variables, upon which girls score as well as boys. Perhaps all we have here is an example of a random significant difference, not improbable at all when so many statistical tests are undertaken, but perhaps the van Hiele test has uncovered some sex difference we do not understand. Another explanation is that the boys (for some unknown reason) may remember the fall test better than the girls; this explanation seems rather less than satisfactory also.

There is a tendency for a greater percentage of boys to be in higher track classes, as can be calculated from the data given in Table 39. There is also a tendency for girls in schools with three tracks of geometry to be in the middle track. However, chi-square analyses of the parts of these tables separated by the dashed lines indicate that these differences are not statistically significant.

CHAPTER VIII: SUMMARY AND CONCLUSIONS

The van Hiele level theory, developed by Pierre Marie van Hiele and his late wife Dina van Hiele-Geldof in the 1950's, is an elegant theory regarding the acquisition of an understanding of geometry as a mathematical system. The theory attempts to explain why many students have difficulty with geometry and what could be done to alleviate these difficulties. It has been applied in curricula in the Netherlands and the Soviet Union and has many adherents in the United States.

There are three aspects to the theory: the existence of levels of understanding in geometry, properties of these levels, and principles underlying the movement from one level to the next. These aspects have been described by the van Hieles in both general and behavioral terms.

The levels (as described by Alan Hoffer) are:

1. Recognition: The student can learn names of figures and recognizes a shape as a whole.
2. Analysis: The student can identify properties of figures.
3. Order: The student can logically order figures and relationships, but does not operate within a mathematical system.
4. Deduction: The student understands the significance of deduction and the roles of postulates, theorems, and proof.
5. Rigor: The student understands the necessity for rigor and is able to make abstract deductions.

The properties of the levels (adapted from P.M. van Hiele) with our names are:

1. Fixed Sequence: A person cannot be at level n without having gone through level n-1.

2. Adjacency: At each level of thought what was intrinsic in the preceding level becomes extrinsic in the current level.
3. Distinction: Each level has its own linguistic symbols and its own network of relationships connecting those symbols.
4. Separation: Two persons who reason at different levels cannot understand each other.
5. Attainment: The learning process which leads to complete understanding at the next higher level has five phases: inquiry, directed orientation, explanation, free orientation, integration.

The phases of movement from one level to the next were not studied in the research summarized here.

The Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project was designed to address a number of questions relating the levels and their properties, particularly regarding the relationships the levels might have with achievement on standard secondary school geometry content.

The CDASSG sample consists of 2699 students in 99 classes in 13 schools in 5 states. All students enrolled in a one-year geometry course in these schools were part of this study. A proof subsample consists of 1520 students in 74 of these classes in 11 schools in these states.

All classes in the CDASSG sample took two tests during the first week of the 1980-81 school year, a test of entering geometry knowledge and a van Hiele level test designed to determine the van Hiele level (if any) of each student. From three to five weeks before the end of the 1980-81 school year, these classes took the van Hiele level test again and also took a standardized test of geometry knowledge. The proof subsample took an additional proof test whose three forms were alternated among the students in these classes. The subsample of students who took all tests but the proof test numbers 1596; 1127 of these students took all five tests.

Testability of the theory. Direct quotes from writings of the van Hieles were used to suggest items for the 25-item van Hiele level test. The quotes at level 5 were found to be quite general or capable of more than one interpretation. As a result, dispute occurred in the project regarding what kinds of questions accurately reflected the intent of the van Hieles. The project's confusion substantiates P.M. van Hiele's disavowal of the existence of this level in his more recent writings. Still, because many of those who use the theory include this level, a test was constructed with all five levels involved. According to this test, level 5 does not act as it should in the theory, often being easier to attain than level 4 and occasionally being easier than level 3.

> Conclusion 1: In the form given by the van Hieles, level 5 either does not exist or is not testable. All other levels are testable.

Ability to classify students into levels. A student was deemed to be at van Hiele level n if the student correctly answered a fixed percentage of questions at level n and at all lower levels. Two criterion percentages were used, 3 of 5 and 4 of 5. The 3 of 5 criterion minimizes the chance of missing a student and yields an optimistic picture of students' levels; the tougher 4 of 5 criterion minimizes the chance of a student being at a level by guessing. The theory with level 5 is called the classical theory and without level 5 is called the modified theory. Thus four different ways of assigning levels to students were used: C3 (classical van Hiele theory, 3 of 5 criterion); C4 (classical, 4 of 5 criterion), M3 (modified van Hiele theory, 3 of 5 criterion), and M4 (modified, 4 of 5).

It was found possible to classify 68% to 92% of students into a van Hiele level, the percentages varying among the criteria as in the table below.

Percentage of students classifiable into
a van Hiele level

Assignment criterion	Fall	Spring
C3	71	68
C4	88	79
M3	85	87
M4	92	86

These percentages tend to verify the fixed sequence property of the van Hiele theory.

Conclusion 2: Over two-thirds and perhaps as many as nine-tenths of students respond to test items in ways which make it easy to assign them a van Hiele level.

However, only about half of the students with van Hiele levels by both the C3 and C4 criteria were assigned the same level.

Conclusion 3: Arbitrary decisions regarding the number of correct responses necessary to attain a level can affect the level assigned to many students.

It is possible that students at level n on the C3 or M3 criterion and at level n-1 on the C4 or M4 criterion are in the process of transition from one level to the next.

Changes in level from fall to spring. A student who did not reach the criterion on level 1, the lowest level, was assigned "level 0". On all assignment criteria in the fall, there are students at level 0 and the plurality of students is at level 1. On all criteria, over half of students classifiable into a van Hiele level are at levels 0 or 1. In the spring, the plurality is at level 3 on the C3 or M3 criterion and at level 2 on the C4 or M4 criterion, and over 40% of those classifiable are below level 3. There is great variability in levels, more so in the spring than in the fall. For instance, less than a third of those students in spring are at the plurality level on the C3, C4, or M4 criterion. This conforms with great variability in the amount of change in van Hiele levels from fall to spring: about a third of students stay at the same level or go down (!), about a third go up one level, and about a third go up two or more levels.

> Conclusion 4: Considering those students at a given van Hiele level in fall, there is great variability in the change in van Hiele level from fall to spring.

That is, as one would expect, there are factors other than van Hiele level operating to contribute to growth in understanding in geometry.

Relationship of van Hiele level to concurrent geometry achievement. The van Hiele level correlates about .60 with knowledge at the beginning of the year, about .64 with performance on a standardized test at the end of the year, and about .63 with proof-writing performance at the end of the year. The C3 criterion generally correlates highest. All correlations are statistically significant at the .0001 level. As high as these correlations are, the correlations between scores on the standardized test and the proof test average even higher at .73.

Conclusion 5: Van Hiele level is a very good predictor of concurrent performance on multiple-choice tests of standard geometry content. Van Hiele level is also a very good predictor of concurrent performance on a proof test, but a content test correlates even higher with proof.

Differences in van Hiele level in fall consistently signal differences in entering knowledge for all C3 and M3 levels below 4 and for all C4 and M4 levels below 3. In spring, differences in van Hiele level consistently signal different performance on an achievement test except for levels 0 and 1 on the C3 and M3 criteria and levels 4 and 5 on the C4 criterion. Differences in van Hiele level do not as consistently indicate differences in proof performance except between levels 2 and 3. However, crosstabulations of spring van Hiele level and number of proofs correct show the following:

a student is quite likely ($p > \frac{2}{3}$) to experience failure in proof	iff	C3 or M3 level < 3 C4 or M4 level < 2
a student has about an even chance of success or failure in proof ($.4 < p < .6$)	iff	C3 or M3 level = 3 C4 or M4 level = 2
a student is quite likely ($p > .7$) to experience success in proof	iff	C3 or M3 level > 3 C4 or M4 level > 2
a student will almost surely ($p > .87$) experience success in proof	iff	C3 or M3 level = 5 C4 or M4 level > 4

Conclusion 6: A C3 or M3 van Hiele level of 3 or a C4 or M4 van Hiele level of 2 acts as a guidepost regarding concurrent success in proof. Above these levels, success in proof is likely. Below these levels, failure in proof is just as likely.

Recall that these guidepost levels are the plurality levels in spring for each of the four criteria.

> Conclusion 7: In geometry classes that have studied proof, the van Hiele levels of most students toward the end of the school year are too low to afford a high likelihood of success in geometry proof.

Relationship of van Hiele level to future geometry achievement. Fall van Hiele level correlates about .51 both with achievement on a standardized geometry test in the spring and with proof-writing ability for those who have studied proof. Though these correlations are statistically significant at the .0001 level, they are about .10 to .15 lower than the corresponding correlations with fall van Hiele level replaced either by spring van Hiele level or by entering geometry knowledge. Again the C3 assignment criterion generally correlates highest. ANOVA indicates that van Hiele level contributes about .10 to the variance in spring geometry achievement after entering geometry knowledge has been considered.

> Conclusion 8: Fall van Hiele level is a good predictor of spring achievement in geometry, but not as good a predictor as either entering knowledge or spring van Hiele level.

Though fall van Hiele level does not correlate as well as entering knowledge with spring achievement, differences in van Hiele levels generally indicate significant differences in likely future performance both on a standardized test and in proof-writing ability, with the only violations of this pattern occurring among levels 3, 4, and 5. These differences tend to disappear when entering knowledge is accounted for, except between levels 2 and 3 on criteria C3 and M3. Together, fall van Hiele level and entering geometry knowledge account for less than half of the variance in spring achievement scores, so there are quite significant intervening variables.

Despite these relatively low contributions to the variance in spring achievement, the probabilities of success in proof are quite a bit higher for those at higher levels than for those at lower levels. The crosstabulations between number of proofs correct and fall level show:

a student is likely ($p > .65$) not to experience success in proof	iff	C3 or M3 level ≤ 1 C4 or M4 level $= 0$
a student has about an even chance of success or nonsuccess in proof ($.38 < p < .60$)	iff	C3 or M3 level $= 2$ C4 or M4 level $= 1$
a student is quite likely ($p > .75$) to experience success in proof	iff	C3 or M3 level ≥ 3 C4 or M4 level ≥ 2

What is noteworthy about these probabilities is that less than 20% of students enter at van Hiele levels high enough to give a likelihood of success in proof. Over 38% of students in the sample and over half of those with van Hiele levels on criterion C3 or M3 are at levels 0 or 1, and so great numbers of students have a high probability of nonsuccess in proof.

> Conclusion 9: In geometry classes that study proof, the fall van Hiele levels of over half the students are too low to afford even a 2 in 5 chance of success at proof.

Taken together, conclusions 7 and 9 support the claims of Izaak Wirszup and Alan Hoffer that many if not most students in the United States enter geometry at van Hiele levels that are too low to insure success and that the geometry course, as presently taught, does not improve their understanding (as measured by van Hiele levels) enough to get that success.

These conclusions can be confirmed by working backwards from those students who perform poorly in spring. Approximately 74% of those students whose standardized test scores were below the 20th percentile on national norms (comprising about a quarter of the students in this study) entered with van Hiele levels of 0 or 1 on the easier C3 criterion. Only 44% of the rest of the population had these levels. Similarly, students who could do no proofs in spring were about twice as likely as the others to have fall van Hiele levels of 0 or 1 on the C3 criterion or 0 on the C4 criterion.

Many schools, recognizing the difficulties that students have with proof, have instituted geometry courses in which proof is studied little if at all. Fourteen of the 99 classes in this study fit that description. Given the probabilities of success, students with C3 van Hiele levels of 0 or 1 should not be placed in a proof-oriented course, while those with van Hiele levels of 3, 4, or 5 (and possibly as low as 2) should not be placed in a non-proof course. In our sample, 44% of students for whom a C3 van Hiele level could be assigned were placed incorrectly into a proof-oriented course and 4% were placed incorrectly into a non-proof course.

Conclusion 10: Using van Hiele levels as the criterion, almost half of geometry students are placed in a course in which their chances of being successful at proof are only 50-50.

The percentage of misplaced students is highest, as one would expect, in schools in which there is not a non-proof alternative. Though the sample of schools is too small to allow definitive conclusions, the data suggest that the offering of a non-proof alternative to the standard geometry course may cut the potential misplacements in half.

The sample of schools is also too small to allow conclusions to be made regarding relationships between socio-economics of the community upon geometry learning and van Hiele levels. However, even this small sample exhibits substantial variation among schools in terms of student performance on the content of a course that is uniformly called "Geometry" and given the same credit towards graduation by all these schools and equally considered by all colleges. In some schools students are more advanced in geometry (as judged by the mean van Hiele level for the school) at the beginning of the year of geometry than students in other schools are at the end of the year.

Student performance in geometry. As the above discussion makes quite clear, many students are quite unsuccessful in geometry. One key factor in this lack of success in the course is the quite poor knowledge of students coming in. For example, in the fall only 52% of students could calculate the area of a square given its sides. No item on the fall test is answered correctly by more than 80% of the students, yet all of the items deal with the simplest of geometry facts and measurement. The best we can say is that the performance of students at the beginning of the geometry course substantiates their need for taking it. Yet approximately 47% of students in the United States do not take geometry, and we must assume that those students tend to be less versed in mathematics than their geometry-taking counterparts.

> Conclusion 11: Many students are not learning even the simplest geometry notions in junior high school; thus many students do not know these notions upon leaving high school.

The CDASSG sample correctly answered about 2% fewer items than the national norming sample on the standardized test, but we do not know if that difference represents an actual decline in performance or is merely a result of a quality difference between the two samples, neither of which was randomly selected. Neither the national norming sample nor the CDASSG sample performs particularly well on some items; e.g., almost half of the students cannot calculate the perimeter of a square given its area and about a fifth of students cannot identify vertical angles.

When considered as a content test, the van Hiele test produces equally depressing results. Whereas virtually all students even in the fall can identify rectangles, over 40% in the spring do not realize that a square can be called a rectangle. One-sixth of students in the spring do not identify a long thin triangle as a triangle. The responses on a question asking about the needs for undefined terms and assumed statements in geometry are almost random.

> Conclusion 12: Many students leave the geometry course not versed in basic terminology and ideas of geometry.

With respect to proof, about 70% of the students who studied proof could do simple proofs requiring only one deduction beyond those made from the given. Thus about 30% cannot do even the simplest proofs. About half of the students who study proof can do proofs requiring longer chains of reasoning. Combining this information with data concerning geometry enrollment yields the following result.

Conclusion 13: Of all high school students in the United States, approximately:

60% do not study proof.	47% do not take geometry
	6% take geometry but drop out before finishing
	7% are enrolled in a non-proof geometry course
40% study proof.	11% study proof but cannot do anything with it
	9% can only do trivial proofs
	7% have moderate success with proof
	13% are successful with proof

Sex differences? Boys score significantly higher than girls (at the .001 significance level) on multiple choice tests of content at the beginning of the year and at the end of the year. The sex differences at the end of the year disappear if the scores are adjusted for entering knowledge.

There are no statistically significant sex differences with respect to the ability to write proofs even without adjusting for entering or concurrent knowledge; with adjustments for knowledge there are still no consistent statistically significant sex differences. The tendency is for raw scores to favor boys and adjusted scores to favor the girls. Geometry proof is a high level task which would seem to make cognitive demands in the areas of spatial reasoning, abstract reasoning, and problem solving. Historically, on tests not associated with school learning, boys have often excelled girls on spatial reasoning and problem solving. The sex equality on such a difficult task as proof suggests that the lack of equality elsewhere may be due to inequality of exposure to the particular ideas being tested.

Conclusion 14: The ability to learn geometry, from facts through proof, is equal between the sexes.

A baffling result concerns sex differences with respect to van Hiele levels. In the fall there are no sex differences, but in the spring there are consistent differences favoring males. This phenomenon runs counter to all other sex difference statistics in the study and we have no explanation for it.

Summary of the summary. It is dangerous to summarize a summary, but we attempt this for the reader who is seeking overall conclusions.

The vast majority of students can be assigned a van Hiele level by a simple test even though the van Hiele level theory has yet to be explicated in a way that enables the testing of its highest level or the assigning to each student a unique level. The levels assigned to students are a good descriptor of concurrent student performance in geometry and a reasonably good descriptor of later performance. The poor performance of many students either on a geometry content test or in proof-writing is strongly associated with being at the lower van Hiele levels. Thus this study confirms the use of the van Hiele level theory to explain why many students have trouble learning and performing in the geometry classroom.

The geometry course is not working for large numbers of students. At the end of their year of study of geometry many students do not possess even trivial information regarding geometry terminology and measurement. Questions regarding mathematical systems are answered in virtual random fashion. Half the students who enroll in a proof-oriented course experience very little or no success with proof. The major cause seems to be lack of knowledge at the beginning of the year. This study confirms the need for systematic geometry instruction before high school if we desire greater geometry knowledge and proof-writing success among our students.

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TABLES 1-39

Table 1. School, tracking policy, class, and student participation in study.

State	STOC[1]	Tracks	Number of Classes	Number of Classes Tested on Proof	Number of Students Taking 1 Test	Number of Students in ALL4 Sample
CA	Low Metro	untracked	4	4	136	47
CA	Main Big City	untracked	9	8	382	214
CA	Main Big City	untracked	4→3[2]	3	138	30
FL	Extreme Rural	untracked	2	2	45	29
FL	Medium City	untracked	3	0	95	64
FL	Medium City	honors regular	2 4 } 6	0 4 } 4	165	73
FL	Medium City	untracked	2	0	35	21
IL	Urban Fringe	honors regular essentials	2 6 } 12 4	2 4 } 6 0	296	230
IL	High Metro/ Low Metro	honors regular empirical	12 11 } 29 6	12 11 } 23 0	757	462
MA	Medium City	honors standard basic	1 3 } 8 4	1 3 } 4 0	192	106
MA	High Metro[4]	honors regular	2 5 } 7	2 5 } 7	147	89
MI	Small Place	untracked	3	3	69	60
MI	Medium City	honors regular	2 9 } 11	2 9 } 11	242	171
OR[3]	Main Big City	untracked	5	5	142	0
Totals (excluding OR)			99	74	2699	1596

[1] Size and type of community

[2] One class was disbanded at the semester; students transferred to others.

[3] Data was collected but test-taking was loosely monitored. Not included in analyses.

[4] Considered as a three-track school due to the existence of a low-level combined algebra and geometry course whose students were not tested in this study.

Table 2. Year in school and age of the CDASSG sample and ALL4 subsample.

Year in School	CDASSG sample		ALL4 subsample	
	n	%	n	%
7	1	0	1	0
8	21	1	16	1
9	241	9	180	11
10	1497	56	962	60
11	685	26	350	22
12	233	9	86	5
	2689	100	1595	100
unknown	10		1	
Total	2699		1596	

Age as of 9/80	n	%	n	%
11	2	0	0	0
12	11	0	9	0
13	63	2	45	3
14	440	16	306	19
15	1258	47	815	51
16	639	24	313	20
17	244	9	96	6
18	26	1	8	1
19	3	0	1	0
20	3	0	3	0
	2678	100	1596	100
unknown	21			
Total	2699			

Table 3. Numbers and percentages of entering geometry students at each classical van Hiele level.

3 of 5 criterion

	VHF subsample		ALL4 subsample	
Level	N	%	N	%
0	140	6	80	5
1	758	32	486	30
2	491	21	337	21
3	201	9	148	9
4	53	2	41	3
5	27	1	20	1
Total fitting	1670	71	1112	70
no fit	691	29	484	30
Totals	2361	100	1596	100

4 of 5 criterion

	VHF subsample		ALL4 subsample	
Level	N	%	N	%
0	708	30	430	27
1	970	41	674	42
2	315	13	209	13
3	84	4	67	4
4	5	0	3	0
5	0	0	0	0
Total fitting	2082	88	1383	87
no fit	279	12	213	13
Totals	2361	100	1596	100

Table 4. Crosstabulation of students fitting classical van Hiele levels in fall with the 3 of 5 criterion and the 4 of 5 criterion, ALL4 subsample.

Classical criteria:

3 of 5	4 of 5: 0	1	2	3	4	5	no fit	Total
0	80	0	0	0	0	0	0	80
1	184	302	0	0	0	0	0	486
2	46	146	117	0	0	0	28	339
3	9	39	38	33	0	0	29	148
4	0	4	9	15	1	0	12	41
5	1	0	1	6	2	0	10	20
no fit	110	183	44	13	0	0	134	484
Total	430	674	209	67	3	0	213	1596

Modified criteria:

3 of 5	4 of 5: 0	1	2	3	4	no fit	Total
0	94	0	0	0	0	0	94
1	216	368	0	0	0	0	584
2	52	190	143	0	0	35	420
3	10	49	52	50	0	38	199
4	1	4	13	23	3	17	61
no fit	67	94	20	0	0	57	238
Total	440	705	228	73	3	147	1596

Table 5. Numbers and percentages of entering geometry students at each modified van Hiele level (level 5 removed from consideration).

3 of 5 criterion

	VHF subsample		ALL4 subsample	
Level	N	%	N	%
0	158	7	94	6
1	900	38	584	37
2	596	25	420	26
3	270	11	199	12
4	80	3	61	4
Total fitting	2004	85	1358	85
no fit	357	15	238	15
Totals	2361	100	1596	100

4 of 5 criterion

	VHF subsample		ALL4 subsample	
Level	N	%	N	%
0	726	31	440	28
1	1008	43	705	44
2	338	14	228	14
3	93	4	73	5
4	5	0	3	0
Total fitting	2170	92	1449	91
no fit	191	8	147	9
Totals	2361	100	1596	100

Table 6. Schematic description and number of students at each level of forced van Hiele assignment, VHF subsample.

	Weighted Sum	Level 1	2	3	4	5	3 of 5 Crit.	Total(%) at level	4 of 5 Crit.	Total(%) at level
Forced VHL0 =	0 C0, M0						140		708	
	2		X				39		87	
	4			X			13		18	
	8				X		6		4	
	16 M0					X	18		18	
	18		X			X	5		3	
	20			X		X	0		2	
	24				X	X	1	222(9)	0	340(36)
Forced VHL1 =	1 C1, M1	X					758		970	
	5	X		X			99		35	
	9	X			X		54		13	
	17 M1	X				X	142		38	
	21	X		X		X	22		3	
	25	X			X	X	10	1085(46)	0	1059(45)
Forced VHL2 =	3 C2, M2	X	X				491		315	
	11	X	X		X		54		10	
	19 M2	X	X			X	105		23	
	27	X	X		X	X	21	671(28)	2	350(15)
Forced VHL3 =	6		X	X			10		8	
	7 C3, M3	X	X	X			201		84	
	22		X	X		X	3		0	
	23 M3	X	X	X		X	69	283(12)	9	101(4)
Forced VHL4 =	13	X		X	X		11		1	
	14		X	X	X		0		0	
	15 C4, M4	X	X	X	X		53		5	
	29	X		X	X	X	2		0	
	30		X	X	X	X	0		0	
	31 C5, M4	X	X	X	X	X	27	93(4)	0	6(0)
Forced No fit=	10		X		X		4		3	
	12			X	X		3		2	
	26		X		X	X	0		0	
	28			X	X	X	0	7(0)	0	5(0)
Total								2361		2361

* An X indicates that the student has met the criterion at that level.

Table 7. Percentages of geometry students entering at van Hiele levels 0 or 1.

	VHF subsample	ALL4 subsample
classical, 3 of 5 criterion as percent of classifiable students	54	51
classical, 3 of 5 criterion as percent of all students	38	35
classical, 4 of 5 criterion as percent of classificable students	81	80
classical, 4 of 5 criterion as percent of all students	71	69

Table 8. Crosstabulation of van Hiele levels in fall and spring, ALL4 subsample.

Classical 3 of 5

Level	S	0	1	2	3	4	5	No fit	TOT
F	0	5	19	13	10	5	1	27	80
	1	15	70	106	109	23	10	153	486
	2	4	22	66	95	36	23	91	337
	3	0	2	7	33	29	44	33	148
	4	0	0	0	6	6	19	10	41
	5	0	0	1	1	2	13	3	20
No fit		7	38	64	88	35	73	179	484
TOT		31	151	257	342	136	183	496	1596

Classical 4 of 5

Level	S	0	1	2	3	4	5	No fit	TOT
F	0	110	121	91	24	1	0	83	430
	1	61	139	181	120	8	5	160	674
	2	4	19	52	68	13	6	47	209
	3	0	0	3	20	18	16	10	67
	4	0	0	0	2	1	0	0	3
	5	0	0	0	0	0	0	0	0
No fit		12	18	43	42	22	10	66	213
		187	297	370	276	63	37	366	1596

Table 8 (continued):

Modified 3 of 5

Level	S	0	1	2	3	4	No fit	TOT
F 0		7	24	21	15	6	21	94
1		18	105	167	168	43	83	584
2		4	31	103	168	78	36	420
3		0	2	14	66	108	9	199
4		0	0	2	16	40	3	61
No fit		5	26	49	83	44	31	238
TOT		34	188	356	516	319	183	1596

Modified 4 of 5

Level	S	0	1	2	3	4	No fit	TOT
F 0		115	126	104	28	1	66	440
1		67	152	213	150	18	105	705
2		4	19	60	90	23	32	228
3		0	0	3	29	37	4	73
4		0	0	0	2	1	0	3
No fit		9	19	30	47	20	22	147
TOT		195	316	410	346	100	229	1596

For each fall level with ten or more students, median spring level is underlined.

Table 9. Percentages of geometry students with indicated changes in van Hiele levels, fall to spring, ALL4 subsample.

	Same or lower	Up 1	Up 2 or more	No fit	Mean increase	N
if begin at level 0:						
Classical 3 of 5	6	24	36	34	1.89	80
Modified 3 of 5	7	26	45	22	1.85	94
Classical 4 of 5	26	28	27	19	1.09	430
Modified 4 of 5	26	29	30	15	1.13	440
if begin at level 1:						
Classical 3 of 5	17	22	29	31	1.26	486
Modified 3 of 5	21	29	36	14	1.23	584
Classical 4 of 5	30	27	20	24	.79	674
Modified 4 of 5	31	30	24	15	.83	705
if begin at level 2:						
Classical 3 of 5	27	28	18	27	.84	337
Modified 3 of 5	33	40	19	9	.74	420
Classical 4 of 5	36	33	9	22	.52	209
Modified 4 of 5	23	39	10	14	.56	228
if begin at level 3:						
Classical 3 of 5	28	20	30	22	.92	148
Modified 3 of 5	41	54	--	5	.47	199
Classical 4 of 5	34	27	24	15	.82	67
Modified 4 of 5	44	51	--	5	.49	73

Table 10. Numbers and percentages of geometry students at each van Hiele level, end of school year.

Classical
3 of 5 criterion

	VHS subsample		ALL4 subsample	
Level	N	%	N	%
0	49	2	31	2
1	223	11	151	9
2	343	17	257	16
3	422	21	342	21
4	162	8	136	9
5	203	10	183	11
Total fitting	1402	68	1100	69
no fit	655	32	496	31
Totals	2057	100	1596	100

Classical
4 of 5 criterion

	VHS subsample		ALL4 subsample	
Level	N	%	N	%
0	266	13	187	12
1	434	21	297	19
2	471	23	370	23
3	335	16	276	17
4	73	4	63	4
5	40	2	37	2
Total fitting	1619	79	1230	77
no fit	438	21	366	23
Totals	2057	100	1596	100

Table 10 (cont'):

Modified
3 of 5 criterion

Level	VHS subsample N	VHS subsample %	ALL4 subsample N	ALL4 subsample %
0	52	3	34	2
1	271	13	188	12
2	470	23	356	22
3	630	31	516	32
4	365	18	319	20
Total fitting	1788	87	1413	89
no fit	269	13	183	11
Totals	2057	100	1596	100

Modified
4 of 5 criterion

Level	VHS subsample N	VHS subsample %	ALL4 subsample N	ALL4 subsample %
0	277	13	195	12
1	455	22	316	20
2	513	25	410	26
3	413	20	346	22
4	113	5	100	6
Total fitting	1771	86	1367	86
no fit	286	14	229	14
Totals	2057	100	1596	100

Table 11. Crosstabulation of weighted sum scores on van Hiele tests in fall and spring, ALL4 subsample, 3 of 5 criterion.

		M0 C0 0	M1 C1 1	2	M2 C2 3	4	5	6	M3 C3 7	8	9	10	11	12	13	14	M4 C4 15	M0 16	M1 17	18	M2 19	20	21	22	M3 23	24	25	26	27	28	29	30	M4 C5 31	TOT
M0 C0	0	5	19	5	13		3	2	10	1	2		2				5	1	4	1	1		1		2		2						1	80
M1 C1	1	15	70	4	106	1	16	5	109	1	2		14	1	2		23	2	17		31		8		35		3		11				10	486
	2	1	5	1	4		1		3				1						1	1					3				1				1	23
M2 C2	3	4	22	1	66		6	2	95		6		10				36		2		18				39		2		4				23	337
	4				1		2		1						1		1				1								1					8
	5	1	9		7		3		16		1						5		2		7		1		9						1		5	67
	6	1			1				3		1														1								1	8
M3 C3	7		2		7		3		33			1	2				29				5		1		20				1				44	148
	8	1					1																											2
	9	1	3		5		1		6				2						1		2				7								3	31
	10																																1	1
	11		1	1	4				7						1				1		5			1	5								8	40
	12						1																											1
	13				2				2								1				1				2						1			9
	14																																	
M4 C4	15								6						1		6				1				6				2				19	41
M0	16	1	1		4				3						1						3													14
M1	17	1	12	1	17	2	4		14				2			1	5		6		13		1	1	10		1		1		1		5	98
	18		2						1												1													4
M2	19		4		12				19				2				7		3	1	7				15				1				12	83
	20																																	
	21		1		1		1		5				1								1		1		3				1				1	16
	22								3																									3
M3	23				1				4						1		8				1				9								27	51
	24				1																													1
	25								1								1				1				2								1	6
	26																																	
	27				2												1								3				2				8	16
	28																																	
	29				2																													2
	30																																	
M4. C5	31				1				1								2								3								13	20
	TOT	31	151	13	257	3	42	9	342	2	12	1	37	1	7	1	136	3	37	3	99		13	3	174		8		26		3		183	1596

Table 12. Crosstabulation of weighted sum scores on van Hiele tests in fall and spring, ALL4 subsample, 4 of 5 criterion.

	M0 C0 0	M1 C1 1	2	M2 C2 3	4	5	6	M3 C3 7	8	9	10	11	12	13	14	M4 C4 15	M0 16	M1 17	18	M2 19	20	21	22	M3 23	24	25	26	27	28	29	30	M4 C5 31	TOT
M0 C0 0	110	121	35	91	3	13	4	24		1		1	1			1	2	4	3	9		3	1	3									430
M1 C1 1	61	139	32	181	5	28	6	120	1	6	1	7		2	1	8	2	11	1	22	1	3		26		1		2		1	1	5	674
2	7	7	2	17			3	12						1		4	1	1	1	2			1	6									65
M2 C2 3	4	19	4	52		5	1	68		2	1	5		2	2	13			2	3		2		15				2		1		6	209
4		2		3	1	2		4													1							1					14
5		3		3		1	1	12								2		1	1	1				2								3	30
6		1						4								2																	7
M3 C3 7				3				20						1		18						1		6						1	1	16	67
8		2				1																											3
9				3				1				3				2		1				1											11
10																1				1													2
11								2				1				2								1									6
12																1																	1
13																																1	1
14																																	
M4 C4 15								2								1																	3
M0 16	3		1	4				1										1															10
M1 17	2	2	2	8		1	1									4	2			2		1	1	4								1	31
18																	1															1	2
M2 19			1	5				3								1						1		4								3	19
20		1																															1
21								1																1									2
22																																	
M3 23								2								2								1								1	6
24																																	
25																																	
26																																	
27																1								1									2
28																																	
29																																	
30																																	
M4 C5 31																																	
TOT	187	297	77	370	9	51	16	276	1	9	2	17	1	6	3	63	8	19	9	40	2	12	3	70		1		5		3	2	37	1596

Table 13. Number of students in each subset of ALL4 used in analyses involving more than one test.*

ALL4:	C3fit	C4fit	M3fit	M4fit
1596	795	1083	1206	1242

Proof Form	ALL5:	C3fit	C4fit	M3fit	M4fit
1	372	175	248	287	288
2	371	189	240	278	281
3	384	200	262	303	309
Totals	1127	564	750	868	878

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*ALL4 students represent 99 classes; ALL5 students represent 74 classes.

Table 14. Pearson correlation coefficients between EG, CAP, VHF and VHS scores and levels.

ALL4:C3 fit	VHF level	VHS level	CAP	
EG	.61	.60	.66	
VHF level		.54	.52	n = 795
VHS level			.65	

ALL4:C4 fit	VHF level	VHS level	CAP	
EG	.58	.62	.64	
VHF level		.55	.51	n = 1083
VHS level			.66	

ALL4:M3 fit	VHF level	VHS level	CAP	
EG	.59	.57	.66	
VHF level		.49	.51	n = 1206
VHS level			.61	

ALL4:M4 fit	VHF level	VHS level	CAP	
EG	.59	.53	.65	
VHF level		.54	.52	n = 1242
VHS level			.67	

All correlations given here are significant at the .0001 level.

Table 15. Pearson correlation coefficients between EG, CAP, VHF, VHS and Proof scores and levels.

ALL5:C3 fit	VHF level	VHS level	CAP	PrfTOT	PrfCOR
EG	.64	.57	.61	.54	.53
	.59	.63	.64	.64	.67
	.62	.52	.65	.55	.55
VHF level		.54	.57	.52	.55
		.50	.52	.56	.57
		.59	.53	.52	.53
VHS level			.62	.63	.55
			.68	.69	.68
			.57	.61	.56
CAP				.76	.73
				.74	.71
				.71	.72
PrfTOT					.91
					.94
					.95

All5:C3 fit sub-sample sizes

Form 1 n = 176
Form 2 n = 189
Form 3 n = 200

ALL5:C4 fit	VHF level	VHS level	CAP	PrfTOT	PrfCOR
EG	.62	.66	.61	.54	.53
	.58	.57	.62	.61	.61
	.52	.56	.61	.51	.47
VHF level		.58	.48	.46	.48
		.52	.57	.50	.51
		.51	.49	.43	.37
VHS level			.69	.63	.62
			.63	.57	.56
			.60	.59	.53
CAP				.73	.70
				.73	.71
				.69	.68
PrfTOT					.90
					.94
					.94

All5:C4 fit sub-sample sizes

Form 1 n = 248
Form 2 n = 241
Form 3 n = 262

Table 15 (continued):

ALL5:M3 fit	VHF level	VHS level	CAP	PrfTOT	PrfCOR
EG	.62 .56 .59	.54 .55 .51	.60 .64 .64	.54 .62 .55	.53 .63 .52
VHF level		.48 .43 .51	.57 .52 .47	.46 .56 .45	.45 .56 .44
VHS level			.56 .60 .57	.57 .62 .58	.53 .58 .51
CAP				.75 .73 .72	.72 .70 .70
PrfTOT					.90 .94 .94

ALL5:M3 fit sub-sample sizes

Form 1 n = 287
Form 2 n = 278
Form 3 n = 303

ALL5:M4 fit	VHF level	VHS level	CAP	PrfTOT	PrfCOR
EG	.61 .57 .54	.64 .58 .59	.60 .65 .64	.54 .62 .54	.54 .61 .50
VHF level		.55 .52 .51	.46 .59 .49	.44 .51 .44	.46 .51 .39
VHS level			.67 .65 .61	.64 .58 .61	.62 .56 .55
CAP				.76 .75 .71	.72 .72 .69
PrfTOT					.90 .94 .94

ALL5:M4 fit sub-sample sizes

Form 1 n = 288
Form 2 n = 281
Form 3 n = 309

Throughout the table the top of the three coefficients refers to form 1 of the Proof test, the middle coefficient refers to form 2, and the bottom coefficient refers to form 3.

All correlations are significant at the .0001 level.

Table 16. Mean EG scores of students at each fall van Hiele level.

Subsample	VHF level	Mean EG	(s.e.)	P(means =)*
ALL4:C3fit	0	6.4	(0.4)	
	1	8.4	(0.2)	.0001
n = 795	2	11.0	(0.2)	.0001
	3	13.6	(0.3)	.0001
	4	15.5	(0.6)	.0026
	5	15.3	(0.8)	.8134
ALL4:C4fit	0	7.6	(0.2)	
	1	9.6	(0.1)	.0001
n = 1083	2	13.1	(0.2)	.0001
	3	15.7	(0.4)	.0001
	4	16.3	(1.8)	.7181
	5	----		-----
All4:M3fit	0	6.9	(0.4)	
	1	8.6	(0.1)	.0001
n = 1206	2	11.0	(0.2)	.0001
	3	13.7	(0.2)	.0001
	4	15.2	(0.4)	.0010
ALL4:M4fit	0	7.7	(0.2)	
n = 1242	1	9.7	(0.1)	.0001
	2	13.3	(0.2)	.0001
	3	15.7	(0.4)	.0001
	4	16.3	(1.8)	.7157

*P(mean at level = mean at next lower level)

Table 17. Mean CAP scores of students at each spring van Hiele level.

subsample	VHS level	mean CAP	(s.e.)	P(means =)*
ALL4:C3fit	0	11.6	(1.1)	
n = 795	1	12.9	(0.5)	.2870
	2	16.1	(0.4)	.0001
	3	20.3	(0.3)	.0001
	4	24.0	(0.5)	.0001
	5	27.3	(0.5)	.0001
ALL4:C4fit	0	12.6	(0.4)	
n = 1083	1	15.0	(0.3)	.0001
	2	19.2	(0.3)	.0001
	3	24.1	(0.3)	.0001
	4	27.6	(0.8)	.0001
	5	29.9	(1.0)	.0856
ALL4:M3fit	0	11.2	(1.1)	
n = 1206	1	13.0	(0.4)	.1147
	2	16.9	(0.3)	.0001
	3	21.3	(0.3)	.0001
	4	26.1	(0.3)	.0001
ALL4:M4fit	0	12.5	(0.4)	
n = 1242	1	15.1	(0.3)	.0001
	2	19.3	(0.3)	.0001
	3	24.7	(0.3)	.0001
	4	29.2	(0.6)	.0001

*P(mean at level = mean at next lower level)

Table 18. Mean PrfTOT and PrfCOR scores of students at each spring van Hiele level.

FORM 1

ALL5 Subsample[§]	VHS level	mean PrfTOT	(s.e.)	P(means =)*	mean PrfCOR	(s.e.)	P(means =)*
C3fit	0	5.2	(2.0)		0.33	(0.41)	
n = 175	1	4.9	(1.2)	.92	0.38	(0.25)	.93
42% TOT	2	9.6	(0.7)	.0014	0.93	(0.15)	.06
31% COR	3	13.4	(0.6)	.0002	1.64	(0.13)	.0006
	4	15.6	(1.0)	.07	2.04	(0.20)	.09
	5	18.7	(0.9)	.021	2.48	(0.19)	.11
C4fit	0	6.0	(0.9)		0.45	(0.17)	
n = 248	1	7.9	(0.7)	.09	0.65	(0.12)	.33
42% TOT	2	12.5	(0.5)	.0001	1.44	(0.10)	.0001
40% COR	3	16.0	(0.7)	.0001	2.17	(0.12)	.0001
	4	18.5	(1.2)	.06	2.59	(0.22)	.09
	5	19.8	(1.6)	.54	2.89	(0.30)	.43
M3fit	0	5.2	(2.1)		0.33	(0.41)	
n = 287	1	5.5	(1.0)	.87	0.42	(0.20)	.85
34% TOT	2	10.7	(0.6)	.0001	1.08	(0.11)	.0048
29% COR	3	14.2	(0.5)	.0001	1.82	(0.10)	.0001
	4	17.3	(0.6)	.0001	2.36	(0.11)	.0004
M4fit	0	6.0	(0.9)		0.47	(0.17)	
n = 288	1	7.9	(0.7)	.08	0.64	(0.12)	.40
42% TOT	2	11.9	(0.5)	.0001	1.38	(0.10)	.0001
40% COR	3	16.6	(0.5)	.0001	2.27	(0.10)	.0001
	4	19.0	(0.9)	.031	2.70	(0.18)	.035

Table 18. (continued)

FORM 2

ALL5 Subsample§	VHS level	mean PrfTOT	(s.e.)	P(means =)*	mean PrfCOR	(s.e.)	P(means =)*
C3fit	0	3.0	(1.8)		0.14	(0.39)	
n = 189	1	8.8	(1.2)	.0086	0.81	(0.26)	.15
39% TOT	2	10.0	(0.8)	.39	0.74	(0.17)	.82
49% COR	3	12.9	(0.6)	.0057	1.45	(0.14)	.0017
	4	17.4	(0.8)	.0001	2.61	(0.17)	.0001
	5	20.7	(0.8)	.0033	3.32	(0.17)	.0031
C4fit	0	8.0	(1.0)		0.75	(0.21)	
n = 240	1	11.0	(0.8)	.018	1.07	(0.17)	.24
35% TOT	2	12.5	(0.6)	.13	1.43	(0.12)	.09
34% COR	3	18.0	(0.6)	.0001	2.66	(0.14)	.0001
	4	19.8	(1.5)	.26	3.15	(0.31)	.15
	5	22.8	(2.6)	.32	3.75	(0.56)	.36
M3fit	0	3.0	(1.9)		0.14	(0.43)	
n = 278	1	8.0	(1.1)	.023	0.73	(0.24)	.24
39% TOT	2	11.2	(0.7)	.013	1.10	(0.15)	.19
35% COR	3	14.7	(0.5)	.0001	1.85	(0.11)	.0001
	4	19.2	(0.5)	.0001	2.99	(0.12)	.0001
M4fit	0	7.8	(0.9)		0.69	(0.21)	
n = 281	1	10.9	(0.8)	.012	1.06	(0.17)	.16
35% TOT	2	12.9	(0.5)	.034	1.49	(0.12)	.037
34% COR	3	18.0	(0.6)	.0001	2.67	(0.13)	.0001
	4	20.9	(1.1)	.022	3.39	(0.24)	.0088

Table 18. (continued)

FORM 3

ALL5 Subsample§	VHS level	mean PrfTOT	(s.e.)	P(means =)*	mean PrfCOR	(s.e.)	P(means =)*
C3fit	0	2.8	(3.1)		0.25	(0.66)	
n = 200	1	5.1	(1.3)	.47	0.38	(0.29)	.86
40% TOT	2	8.7	(0.9)	.026	0.82	(0.19)	.21
34% COR	3	15.2	(0.7)	.0001	2.14	(0.16)	.0001
	4	16.5	(1.2)	.37	2.17	(0.27)	.93
	5	20.2	(1.1)	.025	3.26	(0.24)	.0027
C4fit	0	5.9	(1.1)		0.45	(0.23)	
n = 262	1	7.8	(0.8)	.15	0.82	(0.17)	.19
36% TOT	2	11.5	(0.7)	.0006	1.38	(0.15)	.012
30% COR	3	17.3	(0.7)	.0001	2.42	(0.15)	.0001
	4	20.6	(2.1)	.14	3.33	(0.43)	.048
	5	20.9	(2.3)	.92	3.43	(0.49)	.88
M3fit	0	3.8	(2.7)		.040	(0.60)	
n = 303	1	5.7	(1.1)	.53	0.50	(0.24)	.88
35% TOT	2	9.4	(0.7)	.0052	0.93	(0.16)	.14
27% COR	3	15.1	(0.6)	.0001	2.08	(0.12)	.0001
	4	18.8	(0.7)	.0001	2.84	(0.16)	.0002
M4fit	0	5.7	(1.1)		0.44	(0.22)	
n = 309	1	8.0	(0.8)	.08	0.82	(0.16)	.17
38% TOT	2	11.9	(0.6)	.0001	1.46	(0.13)	.0025
31% COR	3	17.7	(0.6)	.0001	2.53	(0.13)	.0001
	4	21.1	(1.4)	.028	3.47	(0.30)	.0043

*P(mean at level = mean at next lower level)

§Percentages denote the percentage of variance in the dependent variable in the table accounted for by the model

Table 19. Mean PrfTOT and PrfCOR scores of students at each spring van Hiele level, adjusted via ANCOVA for CAP.

FORM 1

ALL5 Subsample[§]	VHS levels	Mean adj. PrfTOT	P(means =)*	Mean adj. PrfCOR	P(means =)*
C3fit	0	9.8 (1.7)		1.27 (0.34)	
n = 175	1	9.0 (1.1)	.67	1.19 (0.22)	.85
63% TOT	2	11.4 (0.6)	.040	1.30 (0.13)	.66
55% COR	3	12.9 (0.5)	.08	1.54 (0.11)	.16
	4	13.5 (0.8)	.57	1.62 (0.17)	.69
	5	15.4 (0.8)	.08	1.81 (0.17)	.40
C4fit	0	9.9 (1.7)		0.94 (0.33)	
n = 248	1	9.9 (1.8)	.99	0.90 (0.15)	.90
59% TOT	2	12.4 (0.5)	.007	1.42 (0.09)	.003
54% COR	3	14.3 (0.7)	.016	1.82 (0.13)	.012
	4	16.2 (2.1)	.40	2.16 (0.41)	.43
	5	16.0 (5.7)	.96	1.06 (1.10)	.35
M3fit	0	10.5 (1.7)		1.32 (0.33)	
n = 287	1	9.7 (0.9)	.65	1.18 (0.17)	.71
51% TOT	2	12.5 (0.5)	.004	1.41 (0.09)	.24
51% COR	3	13.5 (0.4)	.08	1.69 (0.08)	.024
	4	14.7 (0.5)	.051	1.87 (0.10)	.14
M4fit	0	10.3 (0.8)		1.19 (0.16)	
n = 288	1	10.4 (0.6)	.84	1.07 (0.11)	.51
61% TOT	2	12.3 (0.4)	.012	1.43 (0.08)	.011
56% COR	3	14.4 (0.5)	.0010	1.89 (0.10)	.0003
	4	14.6 (0.9)	.89	1.95 (0.17)	.76

Table 19. (continued)

FORM 2

ALL5 Subsample§	VHS levels	Mean adj. PrfTOT	P(means =)*	Mean adj. PrfCOR	P(means =)*
C3fit	0	7.3 (1.6)		0.97 (0.37)	
n = 189	1	12.3 (1.1)	.008	1.49 (0.25)	.21
63% TOT	2	12.1 (0.7)	.89	1.15 (0.17)	.22
61% COR	3	13.3 (0.6)	.20	1.53 (0.12)	.06
	4	15.9 (0.7)	.004	2.30 (0.15)	.0002
	5	17.5 (0.8)	.09	2.70 (0.17)	.07
C4fit	0	11.7 (1.2)		1.25 (0.27)	
n = 248	1	14.6 (0.9)	.06	1.61 (0.20)	.28
58% TOT	2	13.4 (0.5)	.27	1.62 (0.11)	.96
53% COR	3	15.8 (0.7)	.006	2.11 (0.15)	.011
	4	15.6 (2.0)	.94	2.47 (0.45)	.45
	5	22.4 (12.2)	.58	3.44 (2.75)	.73
M3fit	0	7.9 (1.6)		1.18 (0.38)	
n = 278	1	11.8 (1.0)	.034	1.52 (0.22)	.42
59% TOT	2	13.2 (0.6)	.19	1.52 (0.13)	.99
54% COR	3	14.8 (0.4)	.030	1.87 (0.10)	.035
	4	16.6 (0.5)	.005	2.43 (0.12)	.0002
M4fit	0	11.5 (0.8)		1.42 (0.19)	
n = 281	1	13.7 (0.7)	.026	1.62 (0.15)	.38
58% TOT	2	13.9 (0.4)	.78	1.69 (0.10)	.68
54% COR	3	15.4 (0.5)	.034	2.15 (0.12)	.0054
	4	14.8 (1.0)	.60	2.18 (0.23)	.89

Table 19. (continued)

FORM 3

ALL5 Subsample[§]	VHS levels	Mean adj. PrfTOT	P(means =)*	Mean adj. PrfCOR	P(means =)*
C3fit	0	7.8 (2.6)		1.39 (0.55)	
n = 200	1	8.7 (1.2)	.72	1.20 (0.25)	.74
59% TOT	2	10.7 (0.8)	.14	1.27 (0.16)	.80
57% COR	3	14.8 (0.6)	.0001	2.06 (0.13)	.0002
	4	14.2 (1.1)	.64	1.66 (0.22)	.12
	5	16.5 (1.0)	.10	2.42 (0.21)	.011
C4fit	0	10.7 (2.1)		0.93 (0.43)	
n = 262	1	10.1 (0.9)	.80	1.35 (0.18)	.37
56% TOT	2	11.6 (0.6)	.15	1.42 (0.12)	.75
51% COR	3	15.5 (0.8)	.0001	1.99 (0.17)	.006
	4	18.3 (2.2)	.23	2.76 (0.45)	.11
	5	19.5 (3.1)	.76	3.32 (0.64)	.47
M3fit	0	9.4 (2.3)		1.63 (0.50)	
n = 303	1	10.2 (1.0)	.75	1.49 (0.22)	.79
57% TOT	2	11.3 (0.6)	.29	1.35 (0.13)	.57
52% COR	3	14.6 (0.5)	.0001	1.96 (0.10)	.0004
	4	15.6 (0.6)	.16	2.14 (0.14)	.29
M4fit	0	10.0 (1.0)		1.35 (0.21)	
n = 309	1	10.6 (0.7)	.59	1.37 (0.14)	.92
56% TOT	2	12.1 (0.5)	.07	1.51 (0.11)	.45
51% COR	3	14.9 (0.6)	.0005	1.94 (0.12)	.011
	4	17.2 (1.2)	.08	2.65 (0.26)	.011

See notes on page 116.

Table 20. Crosstabulation of spring van Hiele level and PrfCOR.

Form 1

Criterion	VHS	PrfCOR					Total	Mean	% with PrfCOR ≥ 2
		0	1	2	3	4			
C3	0	5	3	0	0	0	8	0.38	0
	1	14	9	2	1	0	26	0.62	12
	2	25	18	13	5	0	61	0.97	30
	3	9	30	18	18	2	77	1.66	49
	4	3	6	10	10	2	31	2.06	71
	5	1	5	20	24	7	57	2.54	89
	Nofit	38	31	28	22	3	112	1.47	47
C4	0	20	8	1	1	0	30	0.43	7
	1	28	23	3	3	0	57	0.67	11
	2	20	27	22	19	0	88	1.45	47
	3	2	15	29	18	4	68	2.10	75
	4	0	1	8	10	3	22	2.68	95
	5	0	0	3	8	1	12	2.83	100
	Nofit	15	28	25	21	6	95	1.74	55
M3	0	5	3	0	0	0	8	0.38	0
	1	15	10	2	1	0	28	0.61	11
	2	31	31	20	9	0	91	1.08	32
	3	14	40	33	33	4	124	1.78	57
	4	4	11	30	34	9	88	2.38	83
	Nofit	16	7	6	3	1	33	0.97	30
M4	0	20	9	1	1	0	31	0.45	6
	1	29	24	3	3	0	59	0.66	10
	2	24	30	25	20	0	99	1.41	45
	3	2	19	36	30	9	96	2.26	78
	4	0	1	11	18	4	34	2.74	97
	Nofit	10	19	15	8	1	53	1.45	45
	Total	85	102	91	80	14	372	1.56	50

Table 20. (continued)

Form 2

Criterion	VHS	PrfCOR					Total	Mean	% with PrfCOR ≥ 2
		0	1	2	3	4			
C3	0	6	1	0	0	0	7	0.14	0
	1	12	4	6	0	0	22	0.73	27
	2	19	21	6	4	0	50	0.90	20
	3	19	25	18	14	9	85	1.64	48
	4	1	10	5	17	12	45	2.64	76
	5	0	2	8	20	29	59	3.29	97
	Nofit	23	28	23	16	13	103	1.69	51
C4	0	18	10	4	0	1	33	0.67	15
	1	18	16	8	6	1	49	1.10	31
	2	25	30	21	18	3	97	1.42	43
	3	2	13	17	23	24	79	2.68	81
	4	1	2	2	6	13	24	3.17	88
	5	0	0	0	2	7	9	3.78	100
	Nofit	16	20	14	16	14	80	1.90	55
M3	0	6	1	0	0	0	7	0.14	0
	1	14	6	6	0	0	26	0.69	23
	2	25	25	10	9	3	72	1.04	31
	3	23	34	30	22	16	125	1.79	54
	4	1	12	13	37	41	104	3.01	88
	Nofit	11	13	7	3	3	37	1.30	35
M4	0	19	10	4	0	1	34	0.65	15
	1	21	17	8	7	1	54	1.07	30
	2	27	31	21	18	7	104	1.49	44
	3	4	14	18	27	30	94	2.69	81
	4	1	2	2	8	20	33	2.42	91
	Nofit	8	17	12	11	4	52	1.73	52
	Tot	80	91	66	71	63	371	1.85	54

Table 20. (continued)

Form 3

Criterion	VHS	PrfCOR					Total	Mean	% with PrfCOR ≥ 2
		0	1	2	3	4			
C3	0	4	2	0	0	0	6	0.33	0
	1	22	5	2	2	1	32	0.59	16
	2	36	12	5	3	3	59	0.73	19
	3	29	10	12	16	23	90	1.93	57
	4	5	6	7	6	14	38	2.47	71
	5	1	1	8	13	22	45	3.20	96
	Nofit	42	16	19	23	14	114	1.57	49
C4	0	24	6	3	1	0	34	0.44	12
	1	38	9	7	6	4	64	0.89	27
	2	37	11	14	10	15	87	1.48	45
	3	9	11	15	25	25	85	2.54	77
	4	0	0	2	3	9	14	3.50	100
	5	0	0	0	4	4	8	3.50	100
	Nofit	31	15	12	14	20	92	1.75	50
M3	0	4	2	0	0	0	6	0.33	0
	1	25	6	2	2	1	36	0.56	14
	2	45	13	7	9	4	78	0.90	26
	3	35	21	19	31	32	138	2.03	59
	4	6	7	15	19	36	83	2.87	84
	Nofit	24	3	10	2	4	43	1.05	37
M4	0	25	6	3	1	0	35	0.43	11
	1	41	10	7	7	4	69	0.88	26
	2	42	11	18	15	17	103	1.55	49
	3	10	13	18	30	34	105	2.62	78
	4	0	0	2	7	13	22	3.50	100
	Nofit	21	12	5	3	9	50	1.34	34
	Tot	139	52	53	63	77	384	1.71	50

Table 21. Mean CAP scores of students at each fall van Hiele level.

ALL4 Subsample	VHF level	mean CAP	(s.e.)	P(means =)*
C3fit	0	14.1	(0.8)	
n = 795	1	16.7	(0.3)	.0035
28%	2	19.8	(0.4)	.0001
	3	24.7	(0.6)	.0001
	4	29.3	(1.1)	.0002
	5	28.3	(1.5)	.60
C4fit	0	14.8	(0.3)	
n = 1083	1	18.8	(0.3)	.0001
27%	2	23.0	(0.5)	.0001
	3	28.5	(0.8)	.0001
	4	31.7	(3.5)	.37
M3fit	0	14.1	(0.7)	
n = 1206	1	17.2	(0.3)	.0001
26%	2	20.5	(0.3)	.0001
	3	25.6	(0.5)	.0001
	4	28.4	(0.8)	.0020
M4fit	0	14.9	(0.3)	
n = 1242	1	19.2	(0.3)	.0001
27%	2	24.0	(0.4)	.0001
	3	28.5	(0.8)	.0001
	4	31.7	(3.6)	.39

*See notes on p. 116.

Table 22. Mean CAP scores of students at each fall van Hiele level, adjusted via ANCOVA for EG.

ALL4 Subsample	VHF level	mean adj. CAP	(s.e.)	P(means =)*
C3fit	0	18.0	(0.7)	
n = 795	1	18.6	(0.3)	.41
47%	2	19.1	(0.3)	.32
	3	21.4	(0.5)	.0002
	4	24.1	(1.0)	.0116
	5	23.3	(1.3)	.64
C4fit	0	17.0	(0.3)	
n = 1083	1	19.0	(0.2)	.0001
44%	2	19.9	(0.4)	.07
	3	22.8	(0.8)	.0003
	4	25.4	(3.0)	.40
M3fit	0	17.7	(0.6)	
n = 1206	1	19.1	(0.3)	.045
46%	2	19.8	(0.3)	.054
	3	22.1	(0.4)	.0001
	4	23.4	(0.7)	.11
M4fit	0	17.3	(0.3)	
n = 1242	1	19.5	(0.2)	.0001
46%	2	20.7	(0.4)	.0077
	3	22.8	(0.7)	.0066
	4	25.3	(3.1)	.44

*See notes on p. 116.

Table 23. Mean PrfTOT and PrfCOR scores of students at each fall van Hiele level.

FORM 1

ALL5 Subsample	VHS level	mean PrfTOT (s.e.)	P(means =)*	mean PrfCOR (s.e.)	P(means =)*
C3fit	0	10.4 (1.8)		1.11 (0.34)	
n = 175	1	9.0 (0.7)	.44	0.85 (0.12)	.47
31% TOT	2	13.2 (0.7)	.0001	1.58 (0.13)	.0001
30% COR	3	17.8 (1.0)	.0001	2.45 (0.18)	.0001
	4	19.2 (2.4)	.60	2.80 (0.45)	.48
	5	19.3 (3.1)	.97	3.00 (0.58)	.79
C4fit	0	8.9 (0.7)		0.87 (0.13)	
n = 248	1	11.2 (0.5)	.012	1.23 (0.09)	.026
22% TOT	2	15.3 (0.8)	.0001	2.06 (0.14)	.0001
24% COR	3	18.9 (1.2)	.013	2.76 (0.22)	.0084
M3fit	0	11.0 (1.6)		1.17 (0.30)	
n = 287	1	10.0 (0.5)	.57	1.03 (0.10)	.66
23% TOT	2	13.7 (0.5)	.0001	1.72 (0.10)	.0001
23% COR	3	17.8 (0.7)	.0001	2.44 (0.14)	.0001
	4	17.9 (1.6)	.95	2.58 (0.30)	.66
M4fit	0	9.2 (0.7)		0.90 (0.13)	
n = 288	1	11.7 (0.5)	.0051	1.31 (0.09)	.011
20% TOT	2	15.4 (0.8)	.0001	2.14 (0.14)	.0001
22% COR	3	18.8 (1.1)	.015	2.69 (0.21)	.026

Table 23. (continued)

FORM 2

ALL 5 Subsample	VHS level	mean PrfTOT (s.e.)	P(means =)*	mean PrfCOR (s.e.)	P(means =)*
C3	0	6.3 (1.6)		0.50 (0.34)	
n = 189	1	11.5 (0.6)	.0028	1.20 (0.14)	.06
34% TOT	2	14.5 (0.7)	.0021	1.85 (0.16)	.0027
34% COR	3	18.5 (0.9)	.0014	2.76 (0.21)	.0006
	4	22.5 (1.9)	.06	3.88 (0.42)	.017
	5	20.4 (1.8)	.94	3.33 (0.39)	.35
C4	0	9.7 (0.7)		0.98 (0.15)	
n = 240	1	14.0 (0.5)	.0001	1.71 (0.11)	.0001
26% TOT	2	17.2 (0.9)	.0031	2.54 (0.20)	.0003
27% COR	3	21.9 (1.4)	.0070	3.67 (0.31)	.0025
	4	24.0 (5.6)	.71	4.00 (1.19)	.79
M3	0	8.1 (1.3)		0.76 (0.28)	
n = 278	1	11.5 (0.5)	.014	1.25 (0.11)	.12
32% TOT	2	15.6 (0.6)	.0001	2.05 (0.13)	.0001
32% COR	3	19.0 (0.8)	.0004	2.88 (0.16)	.0001
	4	21.4 (1.2)	.09	3.60 (0.26)	.021
M4	0	9.7 (0.7)		0.96 (0.14)	
n = 281	1	14.2 (0.5)	.0001	1.77 (0.10)	.0001
26% TOT	2	17.7 (0.8)	.0003	2.63 (0.81)	.0001
26% COR	3	21.6 (1.3)	.013	3.61 (0.29)	.0045
	4	24.0 (5.6)	.68	4.00 (1.23)	.76

Table 23. (continued)

FORM 3

ALL5 Subsample	VHS level	mean PrfTOT (s.e.)		P(means =)*	mean PrfCOR (s.e.)		P(means =)*
C3	0	4.8	(2.0)		0.18	(0.42)	
n = 200	1	9.8	(0.8)	.022	1.05	(0.16)	.052
29% TOT	2	14.4	(0.8)	.0001	1.98	(0.16)	.0001
28% COR	3	19.8	(1.2)	.0003	3.03	(0.26)	.0007
	4	19.4	(1.8)	.87	3.23	(0.38)	.67
	5	16.0	(6.7)	.62	3.00	(1.38)	.87
C4	0	7.9	(0.8)		0.81	(0.17)	
n = 262	1	12.3	(0.6)	.0001	1.59	(0.13)	.0002
19% TOT	2	16.2	(1.0)	.0016	2.20	(0.21)	.015
15% COR	3	20.3	(2.1)	.08	3.18	(0.43)	.042
	4	19.5	(4.9)	.89	2.50	(1.01)	.54
M3	0	4.5	(1.9)		0.15	(0.39)	
n = 303	1	11.1	(0.6)	.0008	1.31	(0.13)	.0054
22% TOT	2	14.0	(0.7)	.0011	1.86	(0.14)	.0039
21% COR	3	19.0	(1.0)	.0001	2.92	(0.20)	.0001
	4	18.9	(1.7)	.93	3.13	(0.35)	.61
M4	0	8.3	(0.8)		0.87	(0.16)	
n = 309	1	12.8	(0.6)	.0001	1.66	(0.12)	.0001
19% TOT	2	16.9	(0.9)	.0001	2.40	(0.19)	.0011
16% COR	3	20.4	(1.9)	.10	3.29	(0.39)	.039
	4	19.5	(4.9)	.87	2.50	(1.02)	.47

*See notes on p. 116.

Table 24. Mean PrfTOT and PrfCOR scores of students at each fall van Hiele level, adjusted via ANCOVA for EG.

FORM 1

ALL5 Subsample	VHF level	Mean PrfTOT adj. for EG	P(means =)*	Mean PrfCOR adj. for EG	P(means =)*
C3fit	0	12.7 (1.8)		1.51 (0.34)	
n = 175	1	10.3 (0.7)	.19	1.08 (0.13)	.22
37% TOT	2	12.8 (0.7)	.012	1.51 (0.13)	.026
36% COR	3	16.0 (1.0)	.008	2.12 (0.19)	.007
	4	15.6 (2.4)	.89	2.17 (0.46)	.92
	5	16.8 (3.0)	.75	2.55 (0.57)	.59
C4fit	0	10.43 (0.8)		1.07 (0.15)	
n = 248	1	12.14 (0.9)		1.37 (0.10)	.10
32% TOT	2	14.36 (1.0)	.044	1.81 (0.18)	.032
32% COR	3	16.89 (4.7)	.60	2.54 (0.87)	.40
M3fit	0	13.0 (1.5)		1.53 (0.29)	
n = 287	1	11.6 (0.6)	.36	1.31 (0.11)	.45
33% TOT	2	13.4 (0.5)	.018	1.67 (0.10)	.013
32% COR	3	15.6 (0.8)	.019	2.02 (0.15)	.043
	4	15.0 (1.5)	.75	2.06 (0.30)	.92
M4fit	0	10.8 (0.7)		1.18 (0.13)	
n = 288	1	12.5 (0.5)	.043	1.46 (0.09)	.07
31% TOT	2	13.6 (0.7)	.22	1.83 (0.14)	.028
32% COR	3	14.4 (1.2)	.55	1.95 (0.23)	.60

Table 24. (continued)

FORM 2

ALL5 Subsample	VHF level	Mean PrfTOT adj. for EG	P(means =)	Mean PrfCOR adj. for EG	P(means =)
C3fit	0	9.7 (1.5)		1.30 (0.32)	
n = 189	1	12.9 (0.6)	.040	1.55 (0.13)	.45
47% TOT	2	14.2 (0.7)	.17	1.77 (0.14)	.25
50% COR	3	16.1 (0.9)	.10	2.19 (0.19)	.08
	4	19.0 (1.8)	.13	3.05 (0.38)	.038
	5	17.6 (0.7)	.57	2.68 (0.35)	.46
C4fit	0	11.3 (0.8)		1.22 (0.16)	
n = 241	1	14.3 (0.5)	.001	1.78 (0.10)	.003
42% TOT	2	14.2 (1.2)	.93	1.76 (0.26)	.96
43% COR	3	20.1 (3.7)	.12	2.93 (0.78)	.15
M3fit	0	10.6 (1.2)		1.34 (0.26)	
n = 278	1	12.9 (0.5)	.068	1.57 (0.11)	.41
45% TOT	2	15.2 (0.5)	.003	1.96 (0.11)	.015
46% COR	3	17.0 (0.7)	.037	2.44 (0.16)	.012
	4	18.3 (1.1)	.30	2.92 (0.25)	.09
M4fit	0	11.7 (0.6)		1.39 (0.14)	
n = 281	1	14.5 (0.4)	.0003	1.83 (0.09)	.0073
43% TOT	2	15.3 (0.8)	.33	2.13 (0.17)	.12
42% COR	3	17.3 (1.3)	.17	2.69 (0.28)	.07
	4	18.1 (5.0)	.87	2.75 (1.11)	.96

Table 24. (continued)

FORM 3

ALL5 Subsample	VHF level	Mean PrfTOT adj. for EG	P(means=)	Mean PrfCOR adj. for EG	P(means =)
C3fit	0	8.0 (2.0)		0.83 (0.42)	
n = 200	1	11.2 (0.8)	.12	1.35 (0.16)	.23
37% TOT	2	14.0 (0.8)	.016	1.90 (0.16)	.02
36% COR	3	17.3 (1.3)	.024	2.53 (0.27)	.04
	4	16.4 (1.9)	.66	2.61 (0.39)	.84
	5	11.5 (6.4)	.46	2.09 (1.32)	.70
C4fit	0	9.5 (1.0)		1.06 (0.20)	
n = 262	1	12.4 (0.6)	.0086	1.62 (0.12)	.015
30% TOT	2	13.5 (1.4)	.46	1.68 (0.30)	.85
26% COR	3	15.7 (3.8)	.59	2.19 (0.77)	.54
	4	66.4 (48.5)	.30	18.39 (9.97)	.11
M3fit	0	8.4 (1.8)		0.91 (0.38)	
n = 303	1	12.7 (0.6)	.019	1.61 (0.13)	.071
34% TOT	2	13.6 (0.6)	.27	1.78 (0.13)	.37
31% COR	3	16.2 (1.0)	.021	2.38 (0.21)	.013
	4	15.0 (1.6)	.48	2.37 (0.35)	.99
M4fit	0	10.4 (0.8)		1.27 (0.16)	
n = 309	1	13.1 (0.5)	.002	1.73 (0.11)	.015
33% TOT	2	14.1 (0.9)	.38	1.84 (0.19)	.63
28% COR	3	16.8 (1.8)	.15	2.60 (0.37)	.06
	4	15.1 (4.6)	.72	1.64 (0.95)	.34

*See notes on p. 116.

Table 25. Crosstabulation of VHF with PrfCOR, ALL5 subsample.

FORM 1

Criterion	VHF	PrfCOR					Total	Mean	% with PrfCOR ≥ 2
		0	1	2	3	4			
C3	0	3	7	1	1	0	12	1.00	16
	1	42	30	17	11	1	101	1.00	29
	2	13	28	21	19	2	83	1.63	51
	3	1	2	15	16	3	37	2.49	92
	4	1	1	1	3	1	7	2.29	71
	5	0	0	2	1	2	5	3.00	100
	Nofit	25	34	34	29	5	127	1.65	54
C4	0	31	26	20	4	0	81	0.96	30
	1	46	51	33	24	3	157	1.28	38
	2	6	9	18	25	3	61	2.16	75
	3	0	1	10	10	4	25	2.68	96
	4	0	0	0	0	0	0	----	--
	5	0	0	0	0	0	0	----	--
	Nofit	2	15	10	17	4	48	2.12	65
M3	0	5	8	2	1	0	16	0.94	19
	1	48	39	22	13	1	123	1.02	29
	2	16	35	26	27	4	108	1.70	53
	3	2	5	20	23	5	55	2.44	87
	4	1	1	3	4	3	12	2.58	83
	Nofit	13	14	18	12	1	58	1.55	53
M4	0	31	26	20	4	0	81	0.96	30
	1	46	54	35	26	3	164	1.30	39
	2	7	9	19	28	4	67	2.19	76
	3	0	2	10	11	4	27	2.63	93
	4	0	0	0	0	0	0	----	--
	Nofit	1	11	7	11	3	33	2.12	64
TOT		85	102	91	80	14	372	1.56	50

Table 25. (continued)

FORM 2

Criterion	VHF	PrfCOR					Total	Mean	% with PrfCOR ≥ 2
		0	1	2	3	4			
C3	0	9	9	1	1	0	20	0.70	10
	1	33	33	22	10	5	103	1.23	36
	2	12	17	20	11	8	68	1.79	57
	3	2	6	4	16	14	42	2.80	81
	4	0	0	0	2	10	12	3.83	100
	5	1	0	0	2	6	9	3.33	89
	Nofit	23	26	19	29	20	117	1.97	58
C4	0	35	24	16	7	0	82	0.94	28
	1	32	47	31	30	17	157	1.70	50
	2	3	6	8	13	15	45	2.69	80
	3	0	0	0	6	12	18	3.67	100
	4	0	0	0	0	1	1	4.00	100
	5	0	0	0	0	0	0	----	---
	Nofit	10	14	11	15	18	68	2.25	65
M3	0	9	9	2	2	0	22	0.86	18
	1	38	40	24	14	5	121	1.24	35
	2	15	22	25	16	15	93	1.93	60
	3	2	6	5	22	17	52	2.88	85
	4	1	0	0	4	16	21	3.62	95
	Nofit	15	14	10	13	10	62	1.82	53
M4	0	37	25	18	8	0	88	0.97	30
	1	33	50	31	30	21	165	1.73	50
	2	4	7	9	15	17	52	2.65	79
	3	0	0	0	8	12	20	3.60	100
	4	0	0	0	0	1	1	4.00	100
	Nofit	6	9	8	10	12	45	2.29	67
TOT		80	91	66	71	63	371	1.85	54

Table 25. (continued)

FORM 3

Criterion	VHF	PrfCOR					Total	Mean	% with PrfCOR ≥ 2
		0	1	2	3	4			
C3	0	14	2	0	0	0	16	0.12	0
	1	55	23	13	11	9	111	1.06	30
	2	27	15	16	21	19	98	1.90	57
	3	7	0	4	11	14	36	2.69	81
	4	1	1	2	1	11	16	3.25	88
	5	0	1	0	1	0	2	2.00	50
	Nofit	35	10	18	18	24	105	1.87	57
C4	0	61	15	9	12	2	99	0.81	23
	1	63	23	26	26	29	167	1.61	49
	2	11	9	10	14	19	63	2.33	68
	3	0	1	2	2	9	14	3.36	93
	4	0	1	0	0	1	2	2.50	50
	5	0	0	0	0	0	0	----	--
	Nofit	4	3	6	9	17	39	2.82	82
M3	0	16	2	0	0	0	18	0.11	0
	1	65	25	17	17	16	140	1.24	36
	2	32	17	21	23	20	113	1.84	57
	3	7	3	6	16	20	52	2.75	81
	4	1	2	2	2	11	18	3.11	83
	Nofit	18	3	7	5	10	43	1.67	51
M4	0	61	15	9	12	3	100	0.81	24
	1	65	23	26	27	32	173	1.64	49
	2	11	10	10	14	20	65	2.34	68
	3	0	1	2	3	9	15	3.33	93
	4	0	1	0	0	1	2	2.50	50
	Nofit	2	2	6	7	12	29	2.86	86
TOT		139	52	53	63	77	384	1.71	50

Table 26. Fail van Hiele levels for those students scoring below 14 on CAP, VHF $\cap$ CAP subsample.

VHF level: C3 criterion

CAP score	0	1	2	3	4	5	Nofit	TOT
2	0	1	0	0	0	0	0	1
3	0	0	0	0	0	0	1	1
4	0	2	0	0	0	0	3	5
5	0	2	0	0	0	0	1	3
6	2	4	5	0	0	0	6	17
7	5	9	3	0	0	0	12	29
8	3	6	4	1	0	0	10	24
9	8	18	5	1	0	0	13	45
10	6	27	10	1	0	0	14	58
11	7	28	9	1	0	0	15	60
12	6	32	13	2	0	0	27	80
13	7	34	14	2	1	0	18	76
TOT	44	163	63	8	1	0	120	399
14 or over	50	366	312	158	45	23	406	1360

VHF level: C4 criterion

CAP score	0	1	2	3	4	5	Nofit	TOT
2	1	0	0	0	0	0	0	1
3	0	0	1	0	0	0	0	1
4	3	0	0	0	0	0	2	5
5	3	0	0	0	0	0	0	3
6	10	6	0	0	0	0	1	17
7	18	8	1	0	0	0	2	29
8	14	8	1	0	0	0	1	24
9	24	13	3	0	0	0	5	45
10	31	20	2	0	0	0	5	58
11	30	25	2	0	0	0	3	60
12	37	30	5	1	0	0	7	80
13	34	30	5	0	0	0	7	76
TOT	205	140	20	1	0	0	33	399
14 or over	272	595	216	73	5	0	199	1360

Table 27. Fall van Hiele levels for students unsuccessful at proof (PrfCOR = 0), ALL5 subsample.

		VHF level							
Criterion	PrfCOR	0	1	2	3	4	5	Nofit	Total
C3	= 0	26	130	52	10	2	1	83	304
	≥ 1	22	185	197	105	33	15	266	823
C4	= 0	127	141	20	0	0	0	16	304
	≥ 1	135	340	149	57	3	0	139	823
M3	= 0	30	151	63	11	3	--	46	304
	≥ 1	26	233	251	148	48	--	117	823
M4	= 0	129	144	22	0	0	--	9	304
	≥ 1	140	358	162	62	3	--	98	823

Table 28. Fall van Hiele levels, C3 criterion, for each track, VHF subsample.

	0	1	2	3	4	5	Nofit	TOT
higher of two	0	3	9	12	10	3	26	63
lower of two	13	95	64	20	2	1	81	276
highest of three	3	44	64	67	25	14	118	335
middle of three	31	221	131	38	7	4	175	607
lowest of three	35	129	49	9	1	1	77	301
untracked	58	266	174	55	8	4	214	779
	140	758	491	201	53	27	691	2361

Table 29. Percentages of students at each van Hiele level in fall, C3 criterion, for different schools using the same text.

Text*	Table School No.	van Hiele level 0	1	2	3	4	5	Nofit
JMS	4	3	30	21	12	3	0	30
	5	6	36	25	6	1	1	27
	6	6	40	23	3	2	0	26
	10	1	32	24	14	1	0	28
	12	2	26	19	15	3	1	35
JG	3	6	19	25	9	3	0	38
	4	10	27	13	20	0	3	27
	8	2	34	29	5	1	0	28
	13	3	55	7	7	0	0	28
MD	1	0	5	11	34	11	9	30
	5	0	0	13	13	25	6	44
	7	3	9	17	22	14	3	31
	9	11	34	21	10	0	0	24

*JMS = Jurgensen, Donnelly, Dolciani, Modern School Mathematics - Geometry, 1969 or 1972.

JG = Jurgensen, Donnelly, Maier, Rising, Geometry, 1975 or 1978.

MD = Moise and Downs, Geometry, any edition.

For each text in each school, the median van Hiele level (of those who fit) is underline.

Table 30. School means for fall and spring classical van Hiele levels, entire CDASSG sample.

School	STOC	VHF:C3	VHS:C3	VHF:C4	VHS:C4
1	High Metro	2.49	3.69	1.54	2.37
2	Medium City	1.96	2.98	1.03	0.80
3	Medium City	1.83	2.33	1.22	1.60
4	Medium City	1.18	2.62	0.84	1.43
5	Medium City	1.71	2.64	0.99	0.69
6	Small Place	1.70	3.24	1.00	2.25
7	High Metro/ Low Metro	1.65	2.61	0.89	1.71
8	Urban Fringe	1.58	2.85	0.95	1.88
9	Extreme Rural	1.57	2.59	0.63	1.73
10	Medium City	1.56	2.38	0.82	1.31
11	Main Big City	1.50	2.48	0.82	1.69
12	Main Big City	1.36	2.30	0.70	1.46
13	Low Metro	1.00	1.41	0.36	0.95

Table 31. School means for EG, CAP, PrfTOT, and PrfCOR, entire CDASSG sample.

STOC	EG	CAP	HP	PrfTOT1	PrfTOT2	PrfTOT3	PrfCOR1	PrfCOR2	PrfCOR3	Rank in Table 30
High Metro	13.57	23.22	.89	13.56	17.05	14.61	1.59	2.42	1.93	1
Medium City[2]	11.14	17.45	.94	-----	-----	-----	----	----	----	3
Small Place	10.69	19.75	.93	10.57	14.68	11.86	1.10	1.84	1.48	6
Medium City	10.30	16.38	.94	11.64	14.21	13.00	1.43	1.93	1.74	2
Urban Fringe	9.83	19.14	.93	12.45	13.87	12.89	1.61	1.71	1.62	8
High Metro/ Low Metro	9.76	21.31	.86	15.27	16.24	16.85	1.91	2.30	2.52	7
Main Big City	9.59	17.98	.75	10.33	11.24	8.26	1.22	1.16	0.74	11
Main Big City	9.56	16.73	.40	12.00	15.42	13.67	1.33	2.11	1.44	12
Extreme Rural	9.56	17.12	.90	13.00	16.69	14.58	1.46	2.15	1.75	9
Medium City	9.09	18.33	.86	8.08	10.32	7.83	0.86	1.00	0.75	5
Medium City	8.78	15.93	.75	12.95	16.52	16.17	1.73	2.48	2.22	10
Medium City[2]	8.71	15.53	.97	-----	-----	-----	----	----	----	4
Low Metro	6.08	12.63	.67	5.59	5.77	5.76	0.45	0.45	0.33	13
Sample mean	9.72	18.80	.83	12.19	13.95	12.74	1.46	1.80	1.64	

[1] Holding power, as defined in Chapter V.

[2] Did not have any classes taking the proof tests.

Table 32. Percentages correct on comparable EG test items for OSU sample (early 1970s) and EG subsample.

Item	Description	OSU	EG
1	Perpendicular lines	55	36
2	Area of rectangle	80	72
3	Similar figures	72	66
4	Obtuse angle	58	49
5	Linear pair, angle notation	50	26
6	Parallel lines	72	72
7	Circle terminology	71	62
8	Parallel line/transversal terms	70	29
9	Right angle	89	76
11	Equilateral triangle	74	79
12	Properties of parallelogram	51	44
15	Perimeter of parallelogram	56	56
16	Similar triangles	56	75
17	Definition of circle	49	45
18	Area of square	46	52
19	Definition of supplementary angles	47	29

Table 33. Percentages of students correctly answering each item on the CAP.

Item	% correct national norms	% correct CAP sample	Item	% correct national norms	% correct CAP sample
1	.84	.82	21	.63	.66
2	.80	.71	22	.57	.58
3	.60	.51	23	.29	.38
4	.56	.56	24	.35	.32
5	.39	.36	25	.53	.35
6	.73	.73	26	.24	.30
7	.79	.80	27	.46	.46
8	.41	.38	28	.33	.32
9	.74	.70	29	.44	.50
10	.47	.36	30	.53	.56
11	.73	.68	31	.38	.35
12	.35	.29	32	.37	.34
13	.27	.34	33	.39	.33
14	.38	.36	34	.30	.33
15	.56	.53	35	.41	.42
16	.56	.49	36	.46	.45
17	.79	.58	37	.40	.30
18	.52	.54	38	.40	.30
19	.57	.60	39	.47	.43
20	.40	.45	40	.24	.26

Other comparisons

n	2695	2015
mean	19.2	18.8
s.d.	7.1	7.3
K-R 20 reliab.	.89	.85

Table 34. Percentile ranks for each score on CAP.

Score	National norms Percentile rank	CAP sample Percentile rank	Score	National norms Percentile rank	CAP sample Percentile rank
1	0	0	21	59	68
2	0	0	22	65	72
3	0	0	23	71	76
4	1	1	24	76	79
5	1	1	25	79	82
6	1	2	26	82	84
7	2	4	27	84	86
8	4	5	28	87	89
9	6	8	29	90	90
10	8	12	30	92	92
11	11	16	31	95	94
12	14	21	32	96	96
13	18	25	33	97	97
14	22	31	34	98	99
15	28	37	35	98	99
16	34	42	36	99	99
17	40	47	37	99	99
18	44	53	38	99	99
19	48	58	39	99	99
20	53	63	40	99	

Table 35. PrfCOR performance, Prf subsample (from Senk (1982)).

	Form 1		Form 2		Form 3	
PrfCOR	n	%	n	%	n	%
0	132	26	124	24	192	38
1	145	29	114	22	65	13
2	111	22	88	17	76	15
3	99	20	104	20	78	15
4	19	4	78	15	95	19
Total	506	100	508	100	506	100
Mean (± s.d.)	1.46 ± 1.18		1.80 ± 1.41		1.64 ± 1.56	

Table 36. PrfTOT performance, Prf subsample (from Senk (1982)).

PrfTOT Interval	Form 1		Form 2		Form 3	
	n	%	n	%	n	%
0 - 4	93	18	49	10	105	21
5 - 8	60	12	68	13	70	14
9 - 12	77	15	83	16	70	14
13 - 16	116	23	99	19	59	12
17 - 20	119	24	111	22	92	18
21 - 24	41	8	98	19	110	22
Total	506	100	508	100	506	100
Mean (± s.d.)	12.9 ± 6.48		13.95 ± 6.50		12.74 ± 7.59	

Table 37. Mean scores on EG, CAP, and Proof tests by sex.

Test	Females			Males			diff. of means statistically significant?
	n	mean	s.e.	n	mean	s.e.	
EG	1183	9.25	(.11)	1228	10.17	(.11)	.001 level M > F
CAP (ALL4)	762	18.75	(.25)	834	19.92	(.25)	.001 level M > F
CAP Adj. for EG	762	19.40	(.19)	834	19.32	(.19)	n.s.
PrfTOT1	219	12.34	(.43)	234	12.91	(.42)	n.s.
PrfTOT1 Adj. for EG	219	12.87	(.36)	234	12.33	(.36)	n.s.
PrfCOR1	219	1.50	(.08)	234	1.55	(.08)	n.s.
PrfCOR1 Adj. for EG	219	1.61	(.07)	234	1.45	(.07)	n.s.
PrfTOT2	214	13.93	(.44)	240	14.60	(.41)	n.s.
PrfTOT2 Adj. for EG	214	14.65	(.36)	240	13.95	(.34)	n.s.
PrfCOR2	214	1.72	(.10)	240	1.97	(.09)	n.s.
PrfCOR2 Adj. for EG	214	1.88	(.88)	240	1.83	(.08)	n.s.
PrfTOT3	241	13.05	(.49)	216	12.82	(.52)	n.s.
PrfTOT3 Adj. for EG	241	13.63	(.41)	216	12.18	(.43)	.05 level F > M
PrfCOR3	241	1.64	(.10)	216	1.75	(.11)	n.s.
PrfCOR3 Adj. for EG	241	1.75	(.09)	216	1.62	(.09)	n.s.

Table 38. Crosstabulation of van Hiele levels by sex, ALL4 subsample (762 females, 834 males).

Criterion	Level	Fall		chi-square value	Spring		chi-square value
		F	M		F	M	
C3	0	41	39		19	12	
	1	236	250		90	61	
	2	163	174	9.16	115	142	28.75
	3	68	80	6 df	145	197	6 df
	4	11	30	n.s.	47	89	p < .001
	5	12	8		88	95	
	Nofit	231	253		258	238	
C4	0	216	214		111	76	
	1	318	356	7.23	156	141	22.16
	2	96	113	4 df	162	208	6 df
	3	26	41	n.s.	114	162	p < .01
	4	1	2		24	39	
	5	0	0		18	19	
	Nofit	105	108		177	189	
M3	0	49	45		20	14	
	1	288	296	5.71	110	78	33.25
	2	206	214	5 df	175	181	5 df
	3	87	112	n.s.	229	287	p < .001
	4	23	38		135	184	
	Nofit	109	129		93	90	
M4	0	224	216		115	80	
	1	337	368	5.105	163	153	22.21
	2	105	123	4 df	184	226	5 df
	3	28	45	n.s.	141	205	p < .001
	4	1	2		42	58	
	Nofit	67	80		117	112	

Table 39. Sex by track.

Track	ALL4 subsample		CDASSG sample	
	F	M	F	M
Highest of two	20	32	28	48
Lower of two	93	99	155	176
Highest of three	125	143	165	183
Middle of three	208	218	352	328
Lowest of three	82	111	163	201
Untracked	234	231	452	448
	762	834	1315	1384

APPENDIX A

Entering Geometry Student Test

This test is reproduced in its entirety on the next four pages. In the actual administration, the four pages covered both sides of two sheets. An answer sheet and item analysis for the EG subsample follow the test.

Test Number ____________

ENTERING GEOMETRY STUDENT TEST

Directions

Do not open this test booklet until you are told to do so.

This entering geometry student test contains 20 questions. It is not expected that you know everything on this test.

There is a test number in the top right hand corner of this page. Write this number in the corresponding place on your answer sheet.

When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 25 minutes for this test.

Wait until your teacher says that you may begin.

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ENTERING GEOMETRY STUDENT TEST

1. Perpendicular lines

(a) intersect to form four right angles
(b) intersect to form two acute and two obtuse angles
(c) do not intersect at all
(d) intersect to form four acute angles
(e) none of the above

2. The area of a rectangle with length 3 inches and width 12 inches is

(a) 18 sq in
(b) 72 sq in
(c) 36 sq in
(d) 15 sq in
(e) 30 sq in

3. If two figures are similar but not congruent then they

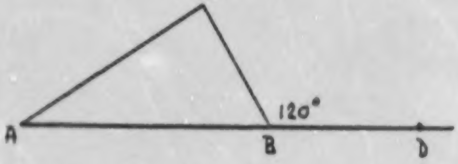
(a) have congruent bases and congruent altitudes
(b) have the same height
(c) both have horizontal bases
(d) have a different shape but the same size
(e) have a different size but the same shape

4. The measure of an obtuse angle is

(a) 90°
(b) between 45° and 90°
(c) less than 90°
(d) between 90° and 180°
(e) more than 180°

5. At right, A, B, and D lie on a straight line. The measure of angle ABC is

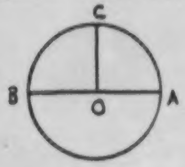
(a) 120°
(b) 60°
(c) 80°
(d) 240°
(e) need more information



6. Parallel lines are lines

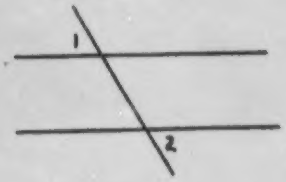
(a) in the same plane which never meet
(b) which never lie in the same plane and never meet
(c) which always form angles of 90° when they meet
(d) which have the same length
(e) none of the above

7. If O is the center of the circle, segment $\overline{OA}$ is called a



- (a) radius of the circle
- (b) diameter of the circle
- (c) chord of the circle
- (d) segment of the circle
- (e) sector of the circle

8. Angles 1 and 2 are called

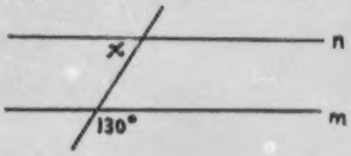


- (a) opposite angles
- (b) parallel angles
- (c) alternate interior angles
- (d) alternate exterior angles
- (e) corresponding angles

9. The measure of a right angle is

- (a) less than 90°
- (b) between 90° and 180°
- (c) 45°
- (d) 90°
- (e) 180°

10. Lines n and m are parallel. The measure of angle x is



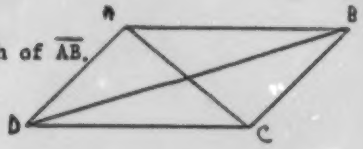
- (a) 65°
- (b) 130°
- (c) 30°
- (d) 40°
- (e) 50°

11. An equilateral triangle has

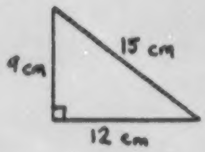
- (a) all three sides the same length
- (b) one obtuse angle
- (c) two angles having the same measure and the third a different measure
- (d) all three sides of different lengths
- (e) all three angles of different measures

12. Given that ABCD is a parallelogram, which of the following statements is true?

- (a) ABCD is equiangular
- (b) Triangle ABD is congruent to triangle CDB.
- (c) The perimeter of ABCD is four times the length of $\overline{AB}$.
- (d) $\overline{AC}$ is the same length as $\overline{BD}$.
- (e) All of the above are true.



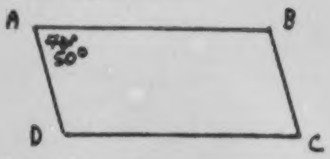
13. The area of the triangle shown is



- (a) 36 sq cm
- (b) 54 sq cm
- (c) 72 sq cm
- (d) 108 sq cm
- (e) 1620 sq cm

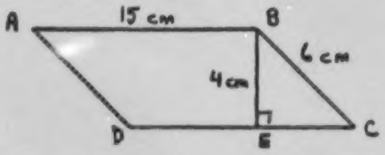
14. ABCD is a parallelogram. The measure of angle C is_

(a) 40°
(b) 130°
(c) 140°
(d) 50°
(e) need more information



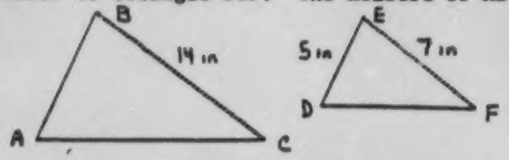
15. The perimeter of this parallelogram ABCD is

(a) 25 cm
(b) 42 cm
(c) 21 cm
(d) 60 cm
(e) 90 cm



16. Triangle ABC is similar to triangle DEF. The measure of $\overline{AB}$ is

(a) 10 in
(b) 11 in
(c) 12 in
(d) 13 in
(e) 15 in

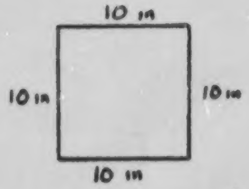


17. The plane figure produced by drawing all points exactly 6 inches from a given point is a

(a) circle with a diameter of 6 inches
(b) square with a side of 6 inches
(c) sphere with a diameter of 6 inches
(d) cylinder 6 inches high and 6 inches wide
(e) circle with a radius of 6 inches

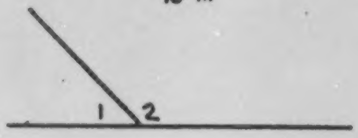
18. The area of the square shown is

(a) 20 sq in
(b) 40 sq in
(c) 40 inches
(d) 100 sq in
(e) 100 inches



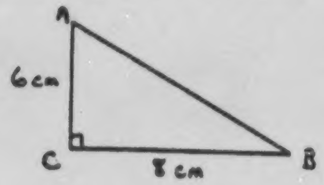
19. Angles 1 and 2 are

(a) interior
(b) vertical
(c) supplementary
(d) complementary
(e) scalene



20. Angle C is a right angle. The length of side $\overline{AB}$ is

(a) 8 cm
(b) 14 cm
(c) 10 cm
(d) 12 cm
(e) 18 cm



Test Number ____________

Project Use Only [✓ S] J
ID ____________________
____

ENTERING GEOMETRY STUDENT TEST
ANSWER SHEET

Please print

Name ____________________________ Class Period ________
Last First Middle

Math Teacher ________________ School ____________________

Grade (circle): 8 9 10 11 12 Sex (circle): M F

Birth date _____ _____ _____
Month Day Year

Testing date _____ _____ _____
Month Day Year

Cross out the correct answer

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E

6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E

11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E

16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E

Space for drawing and figuring
(You may also use the other side)

Entering Geometry Student Test
Item Analysis

Item Number	Percentage with choice						
	A	B	C	D	E	blank	% correct
1	36	14	28	8	11	3	36
2	4	3	72	5	16	0	72
3	5	4	7	16	66	2	66
4	11	20	10	49	9	1	49
5	23	26	6	6	38	1	26
6	72	14	2	5	8	0	72
7	62	10	6	14	7	1	62
8	10	24	7	29	28	2	29
9	5	6	9	76	3	1	76
10	23	37	4	4	30	0	30
11	79	2	12	4	2	1	79
12	5	44	7	9	33	2	44
13	43	24	5	11	15	0	24
14	not used; error in figure						
15	11	56	8	10	13	2	67
16	75	4	16	2	2	1	75
17	17	19	11	6	45	2	45
18	3	19	15	52	10	1	52
19	8	10	29	31	19	3	29
20	9	25	44	16	3	3	44

APPENDIX B

Van Hiele Geometry Test

This test is reproduced in its entirety on the next twelve pages. In the actual administration, the twelve pages covered both sides of six sheets. Identical tests were used in spring and fall. The quotes employed in the construction of items, an answer sheet, and an item analysis follow the test.

Test Number ____________

VAN HIELE GEOMETRY TEST*

Directions

Do not open this test booklet until you are told to do so.

This test contains 25 questions. It is not expected that you know everything on this test.

There is a test number in the top right hand corner of this page. Write this number in the corresponding place on your answer sheet.

When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.

Wait until your teacher says that you may begin.

*This test is based on the work of P.M. van Hiele.

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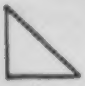
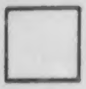
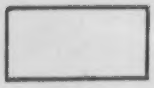
VAN HIELE GEOMETRY TEST

1. Which of these are squares?

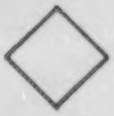
   (A) K only
   (B) L only
   (C) M only
   (D) L and M only
   (E) All are squares.

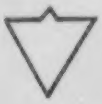

K

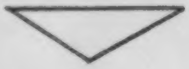

L

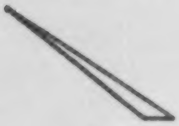

M

2. Which of these are triangles?


U

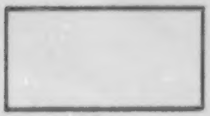

V


W


X

   (A) None of these are triangles.
   (B) V only
   (C) W only
   (D) W and X only
   (E) V and W only

3. Which of these are rectangles?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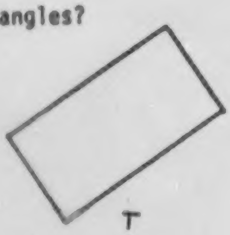
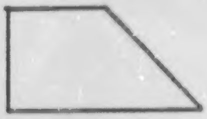
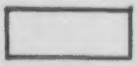
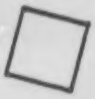

T

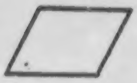

U

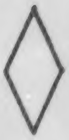
   (A) S only
   (B) T only
   (C) S and T only
   (D) S and U only
   (E) All are rectangles.

4. Which of these are squares?


F

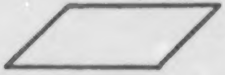

G

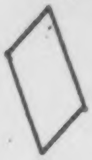

H

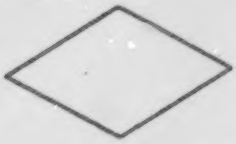

I

(A) None of these are squares.
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares.

5. Which of these are parallelograms?


J

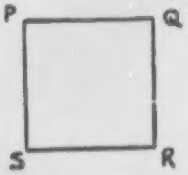

M


L

(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms.
(E) All are parallelograms.

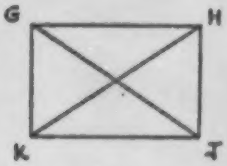
6. PQRS is a square.

Which relationship is true in all squares?



(A) $\overline{PR}$ and $\overline{RS}$ have the same length.
(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.
(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.
(D) $\overline{PS}$ and $\overline{QS}$ have the same length.
(E) Angle Q is larger than angle R.

7. In a rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.

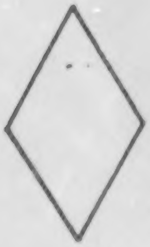
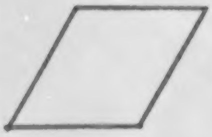


Which of (A)-(D) is not true in every rectangle?

(A) There are four right angles.
(B) There are four sides.
(C) The diagonals have the same length.
(D) The opposite sides have the same length.
(E) All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

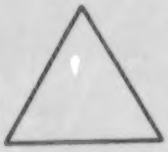
Here are three examples.

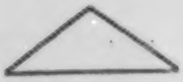



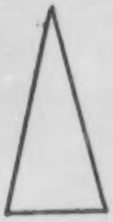
Which of (A)-(D) is not true in every rhombus?

(A) The two diagonals have the same length.
(B) Each diagonal bisects two angles of the rhombus.
(C) The two diagonals are perpendicular.
(D) The opposite angles have the same measure.
(E) All of (A)-(D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.







Which of (A)-(D) is true in every isosceles triangle?

(A) The three sides must have the same length.

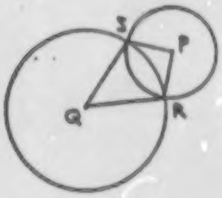
(B) One side must have twice the length of another side.

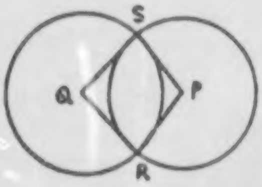
(C) There must be at least two angles with the same measure.

(D) The three angles must have the same measure.

(E) None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.





Which of (A)-(D) is not always true?

(A) PRQS will have two pairs of sides of equal length.

(B) PRQS will have at least two angles of equal measure.

(C) The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.

(D) Angles P and Q will have the same measure.

(E) All of (A)-(D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.
Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.

(B) If 1 is false, then 2 is true.

(C) 1 and 2 cannot both be true.

(D) 1 and 2 cannot both be false.

(E) None of (A)-(D) is correct.

12. Here are two statements.

Statement S: △ABC has three sides of the same length.
Statement T: In △ABC, ∠B and ∠C have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.

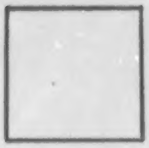
(B) If S is true, then T is true.

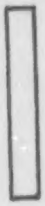
(C) If T is true, then S is true.

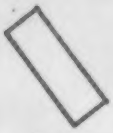
(D) If S is false, then T is false.

(E) None of (A)-(D) is correct.

13. Which of these can be called rectangles?


P


Q


R

- (A) All can.
- (B) Q only
- (C) R only
- (D) P and Q only
- (E) Q and R only

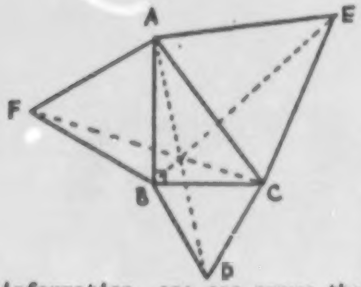
14. Which is true?

- (A) All properties of rectangles are properties of all squares.
- (B) All properties of squares are properties of all rectangles.
- (C) All properties of rectangles are properties of all parallelograms.
- (D) All properties of squares are properties of all parallelograms.
- (E) None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?

- (A) opposite sides equal
- (B) diagonals equal
- (C) opposite sides parallel
- (D) opposite angles equal
- (E) none of (A)-(D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(C) In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(D) In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R.
(B) D implies R which implies S.
(C) S implies R which implies D.
(D) R implies D which implies S.
(E) R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.
II. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove that II is true.

(B) To prove II is true, it is enough to prove that I is true.

(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.

(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.

(E) None of (A)-(D) is correct.

19. In geometry;

(A) Every term can be defined and every true statement can be proved true.

(B) Every term can be defined but it is necessary to assume that certain statements are true.

(C) Some terms must be left undefined but every true statement can be proved true.

(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.

(E) None of (A)-(D) is correct,

20. Examine these three sentences.

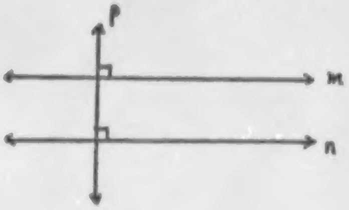
(1) Two lines perpendicular to the same line are parallel.

(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.

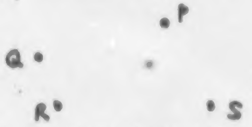
(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)



21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are $\{P,Q\}$, $\{P,R\}$, $\{P,S\}$, $\{Q,R\}$, $\{Q,S\}$, and $\{R,S\}$



Here are how the words "intersect" and "parallel" are used in F-geometry. The lines $\{P,Q\}$ and $\{P,R\}$ intersect at P because $\{P,Q\}$ and $\{P,R\}$ have P in common.

The lines $\{P,Q\}$ and $\{R,S\}$ are parallel because they have no points in common.

From this information, which is correct?

(A) $\{P,R\}$ and $\{Q,S\}$ intersect.
(B) $\{P,R\}$ and $\{Q,S\}$ are parallel.
(C) $\{Q,R\}$ and $\{R,S\}$ are parallel.
(D) $\{P,S\}$ and $\{Q,R\}$ intersect.
(E) None of (A)-(D) is correct.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

    (A) In general, it is impossible to bisect angles using only a compass and an unmarked ruler.

    (B) In general, it is impossible to trisect angles using only a compass and a marked ruler.

    (C) In general, it is impossible to trisect angles using any drawing instruments.

    (D) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.

    (E) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

    > The sum of the measures of the angles of a triangle is less than 180°.

    Which is correct?

    (A) J made a mistake in measuring the angles of the triangle.

    (B) J made a mistake in logical reasoning.

    (C) J has a wrong idea of what is meant by "true."

    (D) J started with different assumptions than those in the usual geometry.

    (E) None of (A)-(D) is correct.

24. Two geometry books define the word rectangle in different ways. Which is true?

(A) One of the books has an error.

(B) One of the definitions is wrong. There cannot be two different definitions for rectangle.

(C) The rectangles in one of the books must have different properties from those in the other book.

(D) The rectangles in one of the books must have the same properties as those in the other book.

(E) The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I. If p, then q.
II. If s, then not q.

Which statement follows from statements I and II?

(A) If p, then s.

(B) If not p, then not q.

(C) If p or q, then s.

(D) If s, then not p.

(E) If not s, then p.

Test Number ____________

VAN HIELE GEOMETRY TEST

ANSWER SHEET

Project Use Only J

ID ______________ ____

Please print

Name ____ Class period ____________
Last First Middle

Math Teacher ____________________ School ______________________________

Grade in School (circle): 8 9 10 11 12 Sex (circle): M F

Birth date ______ ______ ______
Month Day Year

Test date ______ ______ ______
Month Day Year

Cross out the correct answer

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E

Space for drawing or figuring
(You may also use the other side)

Van Hiele Geometry Test
Corresponding Reference Quotes

Item	Level	Quote number	on page*
1	1	1	9
2	1	1	9
3	1	1	9
4	1	8	9
5	1	6	9
6	2	4	10
7	2	4	10
8	2	7	10
9	2	1	10
10	2	3	10
11	3	4	11
12	3	4	11
13	3	6	11
14	3	14	11
15	3	14	11
16	4	6	12
17	4	4	12
18	4	2	12
19	4	2	12
20	4	2	12
21	5	1, 4	12
22	5	4	12
23	5	3	12
24	5	3	12
25	5	2	12

*in this report

Van Hiele Geometry Test
Item Analysis Fall (F) and Spring (S)

Level	Choice	Item	1F	1S	2F	2S	3F	3S	4F	4S	5F	5S
1	A		1	0	4	1	4	1	4	3	9	4
	B		90	89	0	0	1	0	80	79	3	1
	C		1	1	30	17	93	97	4	5	46	15
	D		8	10	63	81	0	0	6	6	5	1
	E		0	0	2	1	2	1	6	7	55	78

2		Item	6F	6S	7F	7S	8F	8S	9F	9S	10F	10S
	A		13	7	14	6	38	69	9	6	16	9
	B		33	66	4	1	18	7	4	2	12	6
	C		40	19	10	7	16	6	69	80	18	9
	D		12	7	7	4	7	3	5	3	38	58
	E		1	1	66	82	20	14	14	8	14	16

3		Item	11F	11S	12F	12S	13F	13S	14F	14S	15F	15S
	A		5	3	19	10	26	56	13	34	14	7
	B		23	15	43	65	2	1	14	16	30	50
	C		48	65	13	10	3	2	21	15	14	7
	D		5	3	7	5	2	1	13	8	16	10
	E		19	13	17	9	68	41	39	26	25	26

4		Item	16F	16S	17F	17S	18F	18S	19F	19S	20F	20S
	A		22	22	31	24	22	21	36	23	24	44
	B		15	12	20	20	21	23	15	23	22	11
	C		33	38	18	28	17	11	23	22	13	5
	D		14	16	22	18	25	33	18	28	33	35
	E		14	11	7	9	14	11	6	3	7	3

5		Item	21F	21S	22F	22S	23F	23S	24F	24S	25F	25S
	A		47	42	14	12	21	29	6	6	13	10
	B		21	29	13	10	12	9	11	9	26	22
	C		11	7	13	9	9	4	21	20	16	8
	D		6	2	28	30	27	41	15	22	30	48
	E		14	18	30	37	29	16	46	42	13	12

The correct choice for each question is underlined.

APPENDIX C

Proof Tests

The same cover sheet was used for each of the three forms of the proof tests, which are reproduced in their entireties on the fifteen pages following the cover sheet. The first five pages constitute Form 1, the next five Form 2, and the third five Form 3. An item analysis follows.

CDASSG GEOMETRY TEST

Name ______________________________ (Last, First) School ____________________

Teacher's name ____________________ Period of day ____________

Your birthdate ___ ___ ___ (Mon. Day Year) Today's date ____________

DIRECTIONS:

1. You will have 35 minutes to complete this test. Take your time but do not spend too much time on any one question.
2. All answers should be written on these pages. If you need more space, use the other side of one of the pages.
3. Work on a question even if you cannot answer it completely, because partial credit will be given.
4. You may use abbreviations for names of theorems. However, each question will be graded by someone other than your teacher. So you should not use names that only your class knows.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

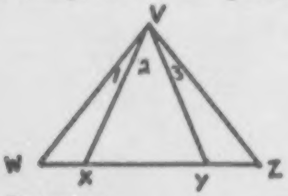
CDASSG GEOMETRY TEST

1. Write statements and reasons to complete this proof.

GIVEN: $\angle W \cong \angle Z$

$\overline{WX} \cong \overline{YZ}$

PROVE: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle W \cong \angle Z$, $\overline{WX} \cong \overline{YZ}$	Given.
2. $\overline{WV} \cong \overline{ZV}$	________________
3. ________________	________________
4. $\angle 1 \cong \angle 3$	________________

2. Statement: If an altitude is drawn to the base of an isosceles triangle, then it bisects the vertex angle.

Suppose you wished to prove the above statement. In the space provided:

1. Draw and label a figure.
2. Write, in terms of your figure, what is given and what is to be proved.

FIGURE:

GIVEN:

TO PROVE:

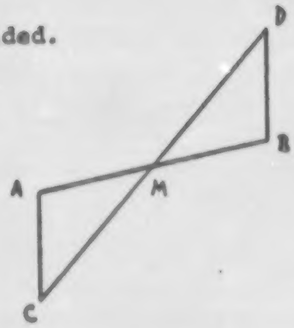
DO NOT PROVE THE STATEMENT. GO ON TO THE NEXT PAGE.

3. Write this proof in the space provided.

GIVEN: M is the midpoint of $\overline{AB}$.

M is the midpoint of $\overline{CD}$.

PROVE: $\triangle ACM \cong \triangle BDM$

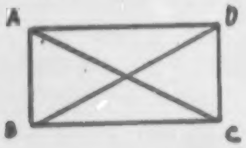


GO ON TO THE NEXT PAGE.

4. Here is a theorem you have had. Complete its proof in the space provided.

Theorem: The diagonals of a rectangle are congruent.

FIGURE:



GIVEN:

TO PROVE:

PROOF:

GO ON TO THE NEXT PAGE.

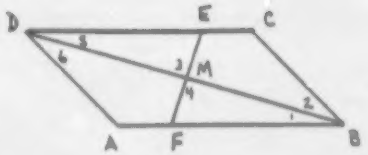
5. Write this proof in the space provided.

GIVEN: AB = DC, AD = BC.

M is the midpoint of $\overline{DB}$.

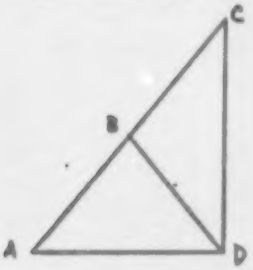
$\overline{EF}$ contains M.

PROVE: FM = ME



GO ON TO THE NEXT PAGE.

6. Write this proof in the space provided.



GIVEN: B is the midpoint of $\overline{AC}$.

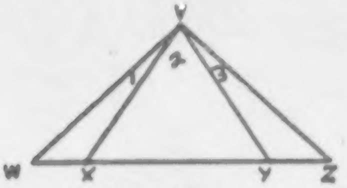
AB = BD.

PROVE: ∠CDA is a right angle.

THIS IS THE LAST PAGE. IF YOU HAVE TIME, YOU MAY GO BACK TO PREVIOUS PAGES.

CDASSG GEOMETRY TEST

1. Write statements and reasons to complete this proof.



GIVEN: $\overline{VW} \cong \overline{VZ}$
$\angle 1 \cong \angle 3$

PROVE: $\triangle VXY$ is isosceles.

Statements	Reasons
1. $\overline{VW} \cong \overline{VZ}$	________________
2. ________________	Base angles of an isosceles triangle are congruent (equal in measure).
3. $\angle 1 \cong \angle 3$	Given.
4. $\triangle VWX \cong \triangle VZY$	________________
5. ________________	Corresponding parts of congruent figures are congruent.
6. $\triangle VXY$ is isosceles.	Definition of isosceles triangle

2. Statement: The diagonals of a rectangle are congruent.

Suppose you wished to prove the above statement. In the space provided:

1. Draw and label a figure.
2. Write, in terms of your figure, what is given and what is to be proved.

FIGURE:

GIVEN:

TO PROVE:

DO NOT PROVE THE STATEMENT. GO ON TO THE NEXT PAGE.

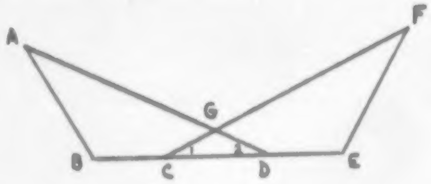
3. Write this proof in the space provided.

GIVEN: $\overline{BD} \cong \overline{EC}$

$\angle 1 \cong \angle 2$

$\angle B \cong \angle E$

PROVE: $\overline{AB} \cong \overline{EF}$



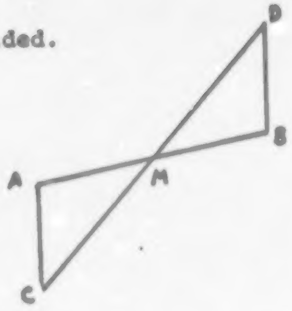
GO ON TO THE NEXT PAGE.

4. Write this proof in the space provided.

GIVEN: M is the midpoint of $\overline{AB}$.

M is the midpoint of $\overline{CD}$.

PROVE: $\overline{AC} \parallel \overline{BD}$

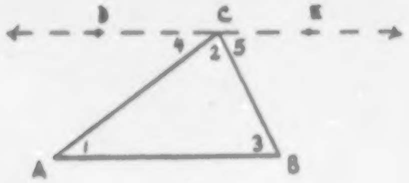


GO ON TO THE NEXT PAGE.

5. This is a theorem you have had. Complete its proof in the space provided.

GIVEN: $\triangle ABC$

PROVE: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statements	Reasons
1. Through point C draw $\overleftrightarrow{DE}$ so that $\overleftrightarrow{DE} \parallel \overleftrightarrow{AB}$.	1.

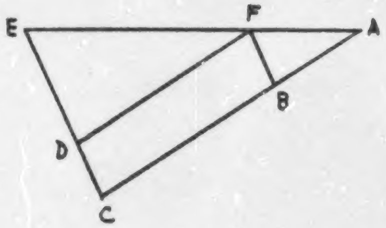
GO ON TO THE NEXT PAGE.

6. Write this proof in the space provided.

GIVEN: $\triangle ABF \sim \triangle ACE$

$\triangle FDE \sim \triangle ACE$

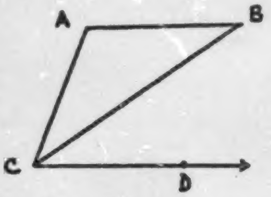
PROVE: BCDF is a parallelogram.



THIS IS THE LAST PAGE. IF YOU HAVE TIME, YOU MAY GO BACK TO PREVIOUS PAGES.

CDASSG GEOMETRY TEST

1. Write statements and reasons to complete this proof.



GIVEN: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$AB = AC$

PROVE: $\overrightarrow{CB}$ bisects $\angle ACD$.

Statements	Reasons
1. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.	Given.
2. $\angle B \cong \angle BCD$.	______________________
3. $AB = AC$	Given.
4. ______________	Base angles of an isosceles triangle are congruent (equal in measure).
5. ______________	Transitive property or substitution
6. ______________	Definition of angle bisector

2. Statement: If a line passes through the midpoints of two sides of a triangle, it is parallel to the third side of that triangle.

Suppose you wished to prove the above statement. In the space provided:

1. Draw and label a figure.
2. Write, in terms of your figure, what is given and what is to be proved.

FIGURE: GIVEN:

TO PROVE:

DO NOT PROVE THE STATEMENT. GO ON TO THE NEXT PAGE.

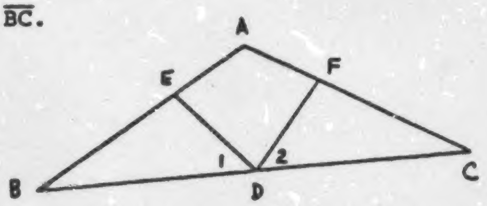
3. Write this proof in the space provided.

GIVEN: D is the midpoint of $\overline{BC}$.

$\angle 1 \cong \angle 2$

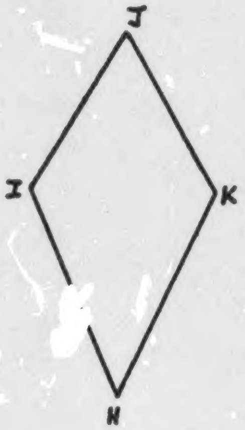
$\overline{DE} \cong \overline{DF}$

PROVE: $\triangle ABC$ is isosceles.



GO ON TO THE NEXT PAGE.

4. Write this proof in the space provided.



GIVEN: Quadrilateral HIJK

HI = HK

IJ = JK

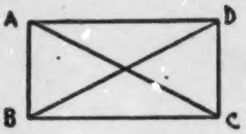
PROVE: $\angle I \cong \angle K$

GO ON TO THE NEXT PAGE.

5. Here is a theorem you have had. Complete its proof in the space provided.

Theorem: The diagonals of a rectangle are congruent.

FIGURE:



GIVEN: ABCD is a rectangle.

TO PROVE: $\overline{AC} \cong \overline{BD}$

PROOF:

GO ON TO THE NEXT PAGE.

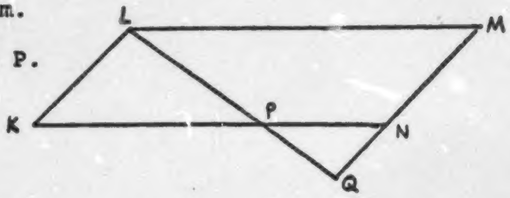
6. Write this proof in the space provided.

GIVEN: KLMN is a parallelogram.

$\overline{LQ}$ and $\overline{KN}$ intersect at P.

N is on line $\overleftrightarrow{MQ}$.

PROVE: $\triangle KLP \sim \triangle NQP$



THIS IS THE LAST PAGE. IF YOU HAVE TIME, YOU MAY GO BACK TO PREVIOUS PAGES.

Proof Tests
Item Analysis

		Percentage Scoring						
Form	Item	0	1	2	3	4	mean	s.d.
1	1	1	8	9	23	35	2.36	1.61
	2	8	8	24	50	10	2.48	1.04
	3	17	8	4	9	63	2.95	1.57
	4	34	15	19	5	26	1.76	1.60
	5	23	20	20	18	19	1.90	1.43
	6	47	41	7	1	5	0.77	0.99
2	1	1	14	15	19	52	3.07	1.14
	2	8	5	10	31	47	3.03	1.21
	3	19	22	5	5	67	2.98	1.61
	4	20	20	12	9	38	2.25	1.60
	5	40	11	7	31	11	1.62	1.52
	6	57	18	7	5	13	0.99	1.42
3	1	11	22	17	24	26	2.32	1.36
	2	10	7	17	20	46	2.86	1.33
	3	31	8	15	10	37	2.14	1.69
	4	40	5	4	4	47	2.14	1.89
	5	37	19	12	8	24	1.63	1.60
	6	34	25	6	12	24	1.66	1.60

APPENDIX D

Test Scripts

Each test administration was accompanied by its own script of two pages length. Appended to each script were answers to questions that students might have or responses to situations that might arise in the test administration. Scripts for all five tests are given here in their entireties followed by the responses.

ENTERING GEOMETRY STUDENT TEST
Teacher Notes and Script

Before you begin this script, a test monitor should have provided you with the following materials:

Enough test booklets, answer sheets, and pencils for the class
A watch or clock (preferably one indicating seconds)
A "testing in progress" sign to be placed outside the door

(Test monitors will have checked all test booklets and answer sheets for correctness and should be consulted in case you have any questions not covered here.)

The directions that follow that are preceded by the word SAY are to be read aloud VERBATIM to the students.

SAY: (name of your school) is one of sixteen high schools selected to participate in a nationwide survey designed to determine what geometry is known by students before they begin their one-year study of geometry. The study involves schools from California, Florida, Illinois, Massachusetts, Michigan, and Oregon. The purpose of the study is to help improve the geometry courses taken by future students.

Today and tomorrow you will be given tests. The scores you make will be used in the study but will not count as part of your grade.

To get an accurate picture of what geometry students know, we must test as many students as possible. However, you have the right to choose not to participate in the survey. Then you must sit quietly while the other students take the tests. If for some reason you do not wish to participate in this survey, raise your hand now.

[Pause.]

Please clear your desks.

DISTRIBUTE THE ANSWER SHEETS AND PENCILS.

SAY: Print your name and school name on the answer sheet. The class period is (give the number or letter). Your math teacher is (give your name). Indicate your grade in school, sex, and birthday. Today's date is (give date). While you are filling in this information, I will distribute the test booklets. Do not open them until you are told to do so.

DISTRIBUTE THE TEST BOOKLETS.

SAY: Follow the directions on the top page as I read them. This entering geometry student test contains 20 questions. It is not expected that you know everything on this test.

There is a test number in the top right hand corner of this page. Write this number in the corresponding place on your answer sheets. [Pause]

When you are told to begin: 1. Read each question carefully. 2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet. 3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet. 4. If you want to change an answer, completely erase the first answer. 5. If you need another pencil, raise your hand. 6. You will have 25 minutes for this test.

If you finish early, check your answers, then sit quietly to allow others to work. Are there any questions? [Pause]

ANSWER QUESTIONS STUDENT HAVE: (You may find a companion sheet helpful.)

You may begin.

WRITE ON BOARD: Test began (time). Test ends (time).

Fifteen minutes later, WRITE ON BOARD: It is now (time).

While students are taking the test, COUNT THE NUMBER TAKING THE TEST AND THE NUMBER OF TEST BOOKLETS YOU HAVE DISTRIBUTED. If these numbers do not agree, notify the test monitor. After exactly 25 minutes:

SAY: Stop. Put your pencils down. This is the end of the test.

Pass your answer sheets (forward) (to the right) (to the left). [Pause.]

Look carefully through your test booklet and erase any pencil marks you find in it. [Pause.] Pass your test booklets in with the directions on top. Pass the pencils in.

Before the students leave, COUNT THE TEST BOOKLETS AND ANSWER SHEETS. Be certain that all test booklets are returned to you.

Please put test booklets and answer sheets in a pile for the test monitor to collect.

Thank you very much for your cooperation. If there is more time, you may handle the class as you wish but please do not try to teach any geometry before tomorrow's test.

VAN HIELE GEOMETRY TEST
Teacher Notes and Script

Before beginning this script, a test monitor should have provided you with the following materials:

Enough test booklets, answer sheets, and pencils for the class
A watch or clock (preferably one indicating seconds)
A "testing in progress" sign to be placed outside the door

(Test monitors will have checked all test booklets and answer sheets for correctness and should be consulted in case you have any questions not covered here.)

The directions that follow that are preceded by the word SAY are to be read aloud VERBATIM to the students.

SAY: Today you will take another test designed for entering geometry students. Please clear your desks.

DISTRIBUTE THE ANSWER SHEETS AND PENCILS.

SAY: Like you did yesterday, print your name and school name on the answer sheet. Your math teacher is [give your name]. The class period is [give the number or letter]. Fill in your grade in school, sex, and birth date. Today's date is [give date]. While you are filling in this information, I will distribute the test booklets. Do not open them until you are told to do so. As soon as you get the test, put the test number on your answer sheet.

DISTRIBUTE THE TEST BOOKLETS.

SAY: Today's test has 25 questions and you will have 35 minutes to answer them. The directions for today's test are identical to yesterday's. Do you have any questions? [Pause.]
If you finish the test early, check your answers, then sit quietly to allow others to work. You may begin.

WRITE ON BOARD: Test began [time]. Test ends [time].

Twenty-five minutes later, WRITE ON BOARD: It is now [time].

While students are taking the test, COUNT THE NUMBER TAKING THE TEST AND THE NUMBER OF TEST BOOKLETS YOU HAVE DISTRIBUTED. If these numbers do not agree, notify the test monitor. After exactly 35 minutes:

SAY: Stop. Put your pencils down. This is the end of the test. Pass your answer sheets (forward) (to the right) (to the left). [Pause.] Look carefully through your test booklet and erase any pencil marks you find in it. [Pause.] Pass your test booklets in with the directions on top. Pass the pencils in.

Before the students leave, COUNT THE TEST BOOKLETS AND ANSWER SHEETS. Be certain that all test booklets are returned to you. Please put all test booklets and answer sheets in a pile for the test monitor to collect.

Thank you very much for your cooperation. If there is more time, you may handle the class as you wish.

VAN HIELE GEOMETRY TEST

Teacher Notes and Script

Before you begin this script, a test monitor should have provided you with the following materials:

Enough test booklets, answer sheets, and pencils for the class
A watch or clock (preferably one indicating seconds)
A "testing in progress" sign to be placed outside the door.

(Test monitors will have checked all test booklets and answer sheets for correctness and should be consulted in case you have any questions not covered here.)

The directions that follow that are preceded by the word SAY are to be read aloud VERBATIM to the students.

SAY: Today and _______ (and _______)* you will be taking [redacted] tests. The scores you make will be used to help us improve the geometry courses taken by future students.

DISTRIBUTE THE ANSWER SHEETS AND PENCILS.

SAY: Fill in all requested information on your answer sheet. Today's date is (give date). While you are filling in this information, I will distribute the test booklets. Do not open them until you are told to do so.

DISTRIBUTE THE TEST BOOKLETS.

SAY: Follow the directions on the top page as I read them.

This test contains 25 questions. It is not expected that you know everything on this test. There is a test number in the top right hand corner of this page. Write this number in the corresponding place on your answer sheet.

When you are told to begin:

1. Read each question carefully.
2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 35 minutes for this test.

Wait until your teacher says that you may begin.

If you finish early, check your answers, then sit quietly to allow others to work. Are there any questions? [Pause]

*For those participating in third day of testing.

ANSWER QUESTIONS STUDENT HAVE: (You may find the companion sheet helpful.)

You may begin.

WRITE ON BOARD: Test began (time). Test ends (time).

While the students are taking the test, we would like for you to fill out the Enrollment Information form. You should receive this from the monitor.

Twenty-five minutes later, WRITE ON BOARD: It is now (time).

While students are taking the test, COUNT THE NUMBER TAKING THE TEST AND THE NUMBER OF TEST BOOKLETS YOU HAVE DISTRIBUTED. If these numbers do not agree, notify the test monitor. After exactly 35 minutes:

SAY: Stop. Put your pencils down. This is the end of the test. Pass your answer sheets (forward) (to the right) (to the left). [Pause.] Look carefully through your test booklet and erase any pencil marks you find in it. [Pause.] Pass your test booklets in with the directions on top. Pass the pencils in.

For Classes with 40-Minute Periods Only

After the materials are collected,

SAY: The second day of testing is (tomorrow) (Wednesday) ( ). A different test will be given which requires 40 minutes. It will be necessary to start the test at the beginning of the period. The directions will be similar to those you had today. Please come early so that you are prepared to start to work as the period begins (tomorrow) (on ____________).

Before the students leave, COUNT THE TEST BOOKLETS AND ANSWER SHEETS. Be certain that all test booklets are returned to you.

Please put test booklets and answer sheets in a pile for the test monitor to collect.

Thank you very much for your cooperation. If there is more time, you may handle the class as you wish but please do not try to teach any geometry before the next test.

COMPREHENSIVE ASSESMENT PROGRAM GEOMETRY TEST

Teacher Notes and Script

Before beginning this script, a test monitor should have provided you with the following materials:

Enough test booklets, answer sheets, and pencils for the class
A watch or clock (preferably one indicating seconds)
A "testing in progress" sign to be placed outside the door

(Test monitors will have checked all test booklets and answer sheets for correctness and should be consulted in case you have any questions not covered here.)

The directions that follow that are preceded by the word SAY are to be read aloud VERBATIM to the students.

SAY: Today you will take another test for geometry students. Please clear your desks.

DISTRIBUTE THE ANSWER SHEETS AND PENCILS.

SAY: Like you did yesterday, fill in the blanks on the answer sheet. Today's date is [give date]. While you are filling in this information, I will distribute the test booklets. Do not open them until you are told to do so. As soon as you get the test, put the test number on your answer sheet. Do not write on the test booklets

DISTRIBUTE THE TEST BOOKLETS.

SAY: The test has 40 questions and you will have 40 minutes to answer them. The directions for today's test are identical to yesterday's. Do you have any questions? [Pause.]
If you finish the test early, check your answers, then sit quietly to allow others to work. You may begin.

WRITE ON BOARD: Test began [time]. Test ends [time].

While students are taking the test, please fill out the Geometry Content Information form. This form is available from the monitor.

Thirty minutes later, WRITE ON BOARD: It is now [time].

While students are taking the test, COUNT THE NUMBER TAKING THE TEST AND THE NUMBER OF TEST BOOKLETS YOU HAVE DISTRIBUTED. If these numbers do not agree, notify the test monitor. After exactly 40 minutes:

SAY: Stop. Put your pencils down. This is the end of the test. Pass your answer sheets (forward) (to the right) (to the left). [Pause.] Look carefully through your test booklet and erase any pencil marks you find in it. [Pause.] Pass your test booklets in with the directions on top. Pass the pencils in.

Before the students leave, COUNT THE TEST BOOKLETS AND ANSWER SHEETS. Be certain that all test booklets are returned to you. Please put all test booklets and answer sheets in a pile for the test monitor to collect.

Thank you very much for your cooperation. If there is more time, you may handle the class as you wish.

CDASSG GEOMETRY TEST

Teacher Notes and Script

Before beginning this script, a test monitor should have provided you with the following materials:

Enough test booklets and pencils for the class
A watch or clock (preferably one indicating seconds)
A "testing in progress" sign to be placed outside the door

(Test monitors should be consulted in case you have any questions not covered here)

The directions that follow that are preceded by the word SAY are to be read aloud VERBATIM to the student.

SAY: Today you will take another test for geometry students.
Please clear your desks. [Pause.]
I will distribute the test booklets. Do not turn the page until you are told to do so.

DISTRIBUTE THE TEST BOOKLETS AND PENCILS.

SAY: Fill in the information on the top page of the test booklet.
Today's date is (give date).
[Pause.]

SAY: Follow the directions on the top page as I read them.

1. You will have 35 minutes to complete this test. Take your time but do not spend too much time on any one question.
2. All answers should be written on these pages. If you need more space, use the other side of one of the pages.
3. Work on a question even if you cannot answer it completely, because partial credit will be given.
4. You may use abbreviations for names of theorems. However, each question will be graded by someone other than your teacher. So you should not use names that only your class knows.

Do not turn this page until you are told to do so.
Do you have any questions? [Pause.]
If you finish the test early, check your answers, then sit quietly to allow others to work. You may begin.

WRITE ON BOARD: Test began [time]. Test ends [time].

While students are taking the test, please fill out the Proof Information Form. This form is available from the monitor.

Twenty-five minutes later, WRITE ON BOARD: It is now [time].

While students are taking the test, COUNT THE NUMBER TAKING THE TEST AND THE NUMBER OF TEST BOOKLETS YOU HAVE DISTRIBUTED. If these numbers do not agree, notify the test monitor. After exactly 35 minutes:

SAY: Stop. Put your pencils down. This is the end of the test. Pass your test booklets (forward) (to the right) (to the left), with the directions on top. [Pause.] Pass the pencils in.

Before the students leave, COUNT THE TEST BOOKLETS. Be certain that all test booklets are returned to you. Please put all test booklets in a pile for the test monitor to collect.

Thank you very much for your cooperation. If there is more time, you may handle the class as you wish.

ANSWERS TO QUESTIONS WHICH MIGHT ARISE DURING TEST ADMINISTRATION

Note: Answers to many possible questions are covered in the teacher script. Questions given here are those not covered there.

May students go to the washroom during the test? Yes, if the teacher agrees and if the test booklet and answer sheet are handed in (not to be returned). Only one student from a class should be allowed out at any time.

What if a student comes in late? If there is enough time in the period to allow the test to be finished, start the student on the test and keep track of time and let the student work the alloted time after everyone else has finished.

If there is not enough time to finish the test, the student should be asked to sit quietly while the others are taking the test.

What should be done with a student who wishes not to take the test? The student should be required to take the test, just as the teacher would require the student to take other tests given by the teacher or the guidance department. (Of course, if there are special circumstances, the teacher should exercise judgment.)

Will students' names be used in any report of the study? Absolutely not. Although schools might be named in a report, students' scores will not be associated with their names. School data will be sent back to the schools, however.

Can the school keep some tests? One copy may be given to schools if so requested but should not be available either to teachers or students until after the school year is over.

A student is absent the first day. What should be done on the second day? The student should take the second day's test.

Should an absent student have to make up the tests? No, we have enough people in the study to make this unnecessary.

Will data for individual classes be offered to schools? We think not. This is not a test of teachers.

What should be done if there is a fire drill or other major disruption during the test? After the disruption is over, collect the tests. Disruptions make data unreliable and so the information is not useful to the study.

How important is it to follow the script? Very important. Few people realize how much what is said before a test affects student performance on the test. By having everyone follow the same script, and one which is designed to be neutral towards the test, we insure scores that are more reliable.

A foreign student is in the class. The student cannot read English well. Can I help this student? Absolutely not. But tell the project representative about the student so that we can note a possible reason for a lower score than usual.

A student does not understand a question. What kind of help can be given the student? None, not even reading the question to the student or pointing out something with your finger.

DO NOT ALLOW ANY TALKING DURING THE TESTS.

DO NOT HELP STUDENTS ANSWER QUESTIONS IN ANY WAY.

APPENDIX E

Monitor Forms

Monitors were given detailed instructions in fall and spring and required to report on each class involved in the study. The complete forms are reproduced here.

INSTRUCTIONS TO MONITORS

Your job: Your job is four-fold:

1. to make it easier for teachers to administer the tests,
2. to insure that the tests are given under uniform conditions like those given in the proctor notes,
3. to be available for assistance in case unexpected questions or situations arise, and
4. to report back to the project on what has transpired in the schools.

Purpose of these instructions: To make it easier for you to do these tasks. The instructions are organized in more or less chronological order. You may wish to check things off as you do them.

BEFORE GOING TO A SCHOOL:

_______ Pick up forms which give you the school name and address, the name of the contact at the school, and which indicate the number of test forms and answer sheets you should have.

_______ Pick up the test package for the school. Check that everything that is supposed to be in the package is actually there. Count individually the number of forms. Do not take for granted that what looks like a package of 35 tests actually has that many tests. We have been known to make errors.

_______ Know the phone number of the project leader in your area so that you can contact him or her in case of emergency. If you cannot contact that person, you may always call 312-753-2616 and give a message to Valerie Payne, the secretary for the project. (To call collect, call 312-753-4167.)

_______ If you are unfamiliar with the school, make certain that you have a map of the area and directions for reaching the school.

AT THE SCHOOL BEFORE TESTING:

First day - The Entering Geometry Student Test.

_______ You are a visitor at the school. Many schools require that you register at an office as a visitor. You must do so if required.

_______ Notify the contact that you are at the school.

_______ Verify that class periods will be at least 35 minutes long. If not, postpone testing until the next day.

_______ If you do not have a schedule of the geometry classes, including periods and room numbers, get one from the department head or the contact.

_______ Before school begins locate the rooms to be used.

_______ Distribute classroom sets of the Entering Geometry Student Tests, answer sheets and pencils to the first teachers who will use them.

Second Day - the Van Hiele Geometry Test

_______ If necessary, register at an office as a visitor.

_______ Notify the contact that you are in the school.

_______ Verify that class periods are at least 40 minutes long.

_______ Locate the rooms you are to monitor that day.

_______ Distribute classroom sets of the Van Hiele Geometry Test, answer sheets and pencils to the first teachers who will use them.

DURING TESTING - BOTH DAYS

If you are monitoring two classes simultaneously, the following instructions apply to both classes.

_______ Notify the teachers that there are scripts which must be followed.

_______ Point out that the last page of the script contains answers to questions that students may ask.

_______ Answer any questions the teacher may have. "I don't know" may be an appropriate response to many questions.

_______ Suggest that the teacher write his or her name and the class period on the board before the period begins.

_______ The teacher is expected to proctor the test. However, in an emergency you should substitute for the teacher.

_______ As the teacher reads the script, assist, if needed, with distribution of materials.

_______ Hang "Testing In Progress" sign on door.

_______ In case of unexpected evacuation (fire drill, bomb scare, etc.) follow school policy, but make every effort to secure the tests. Report such unusual circumstances on the monitor report form.

_______ Record the names of students entering late, and the manner in which the situation was handled on the monitor report form.

*_______ Complete the Monitor Report Form.

_______ At the end of test period, assist, if needed, with collecting the materials.

_______ Arrange the tests and answer sheets in numerical order. Immediately rectify any discrepancies.

_______ Check booklets for pencil marks. Erase any the students may not have caught. Do not use booklet again if marks cannot be erased.

_______ Take down "Testing In Progress" sign.

_______ If time permits, sharpen pencils before next class uses test booklets.

_______ If another class will use booklets, distribute classroom sets of tests, answer sheets and pencils to next teacher.

AFTER TESTING FIRST DAY:

_______ Take home all tests, answer sheets and pencils you brought to the school. Do not leave them in school.

AFTER TESTING SECOND DAY:

1. Complete the School Inventory Form
2. Pick up all Class Information Forms (one per class) from the teachers.
3. Return these forms and the completed Monitor Report Forms to the project leader in your area.

Z. Usiskin
CDASSG Project
8-80

INSTRUCTIONS TO MONITORS

Your job: Your job is four-fold:

1. to make it easier for teachers to administer the tests,
2. to insure that the tests are given under uniform conditions like those given in the proctor notes,
3. to be available for assistance in case unexpected questions or situations arise, and
4. to report back to the project on what has transpired in the schools.

Purpose of these instructions: To make it easier for you to do these tasks. The instructions are organized in more or less chronological order. Please read over the instructions in advance. You may wish to check things off as you do them.

BEFORE GOING TO A SCHOOL:

_______ Pick up forms which give you the school name and address, the name of the contact at the school, and which indicate the number of test forms and answer sheets you should have.

_______ Pick up the test package for the school. Check that everything that is supposed to be in the package is actually there. Count individually the number of forms. Do not take for granted that what looks like a package of 35 tests actually has that many tests. We have been known to make errors.

_______ Know the phone number of the project leader in your area so that you can contact him or her in case of emergency. If you cannot contact that person, you may always call 312-753-2616 and give a message to Valerie Payne, the secretary for the project. (To call collect, call 312-753-4167.)

_______ If you are unfamiliar with the school, make certain that you have a map of the area and directions for reaching the school.

AT THE SCHOOL BEFORE TESTING:

First day - Van Hiele Geometry Test.

_______ You are a visitor at the school. Many schools require that you register at an office as a visitor. You must do so if required.

_______ Notify the contact that you are at the school.

_______ Verify that class periods will be at least 40 minutes long. If not, postpone testing until the next day.

________ If you do not have a schedule of the geometry classes, including periods and room numbers, get one from the department head or the contact.

________ Before school begins locate the rooms to be used.

________ Distribute the classroom sets of the van Hiele Geometry Tests (yellow), Teacher Scripts (yellow), answer sheets and pencils to the first teachers who will use them.

________ Give Enrollment Information Form to teacher to fill out while students are taking the test.

Second Day - Comprehensive Assessment Program (CAP)

________ If necessary, register at an office as a vistor.

________ Notify the contact that you are in the school.

________ Verify that class periods are at least 40 minutes long.

________ Locate the rooms you are to monitor that day.

________ Distribute classroom sets of the CAP Geometry Test, Teacher Scripts (pink), answer sheets and pencils to the first teachers who will use them.

________ Give Geometry Content Information Form to teacher to fill out during the test.

*Third Day (only for those classes taking the CDASSG Geometry Test - the test of proof)

________ If necessary, register at an office as a visitor.

________ Notify the contact that you are at the school.

________ Verify that class periods will be at least 40 minutes long.

________ Locate the rooms you are to monitor that day.

________ Distribute classroom sets of the CDASSG Geometry Test (white), Teacher Scripts (blue), and pencils to the teachers who will use them.

________ Give Proof Information Form to teacher to fill out while students are taking the tests.

DURING TESTING - ALL DAYS

If you are monitoring two classes simultaneously, the following instructions apply to both classes.

________ Notify the teachers that there are scripts which must be followed.

________ Point out that the last page of the script contains answers to questions that student may ask.

________ Answer any questions the teacher may have. "I don't know" may be an appropriate response to many questions.

________ The teacher is expected to proctor the test. However, in an emergency you should substitute for the teacher.

________ As the teacher reads the script, assist, if needed, with distribution of materials.

________ Ask the teacher to fill out the appropriate information form during the test.

________ Hang "Testing In Progress" sign on door.

________ In case of unexpected evacuation (fire drill, bomb scare, etc.) follow school policy, but make every effort to secure the tests. Report such unusual circumstances on the monitor report form.

________ Record the names of students entering late, and the manner in which the situation was handled on the monitor report form.

________ Complete the Monitor Report Form.

________ At the end of test period, assist, if needed, with collecting the materials.

________ Arrange the tests and answer sheets in numerical order. Immediately rectify any discrepancies.

________ Check booklets for pencil marks(first and second day). Erase any the students may not have caught. Do not use booklet again if marks cannot be erased.

________ Take down "Testing in Progress" sign.

________ If time permits, sharpen pencils before next class uses text booklets.

________ If another class will use booklets, distribute classroom sets of tests, answer sheets and pencils to next teacher.

AFTER TESTING EACH DAY:

________ Take home all tests, answer sheets and pencils you brought to the school. Do not leave them in school.

AFTER FINAL DAY OF TESTING

1. Complete the School Inventory Form

2. Return all information forms filled out by teachers and the completed Monitor Report Forms to the project leader in your area.

MONITOR REPORT FORM (One per monitor per day)

Monitor name ______________________ School name ______________________

Date ______________________ Test name ______________________

Period	Teacher name	Total Number of Students	Number taking test	Unusual questions asked by students/Special circumstances during test?

APPENDIX F

Teacher-Completed Forms

On all testing days but the first, teachers were asked to complete forms while their students took the tests. These forms are reproduced here in their entireties.

CLASS INFORMATION FORM
(To be completed by the teacher)

School ______________________________ Teacher Name ____________________

Course Title ______________________________ Period of Day ____________________

Number of times class meets per week ________ Normal length of period (minutes) ______

Approximate number of students enrolled __________________

Number taking Entering Geometry Student Test ____________

Van Hiele Geometry Test ____________

Text(s) which student must have for this course (give title, author, and last copyright dat

____

____

Are there required review books or workbooks? If so, please name.

____

In this course, will students be expected to be able to write proofs? ____________

If so, in what month will they begin to write proofs? ______________________

How would you describe the amount of emphasis on proof in this course?

high? _______ medium? _______ low? _______ none? _______

At the end of the year, we are considering giving students questions dealing with writing proofs similar to those found in standard textbooks. Would you consider such questions to be appropriate for your class? ________________

ENROLLMENT INFORMATION FORM -- Part I

To the Teacher: The purpose of this part is to determine what happened to those students who participated in the fall testing but who are not being tested in the spring.

School ______________________________ Date ____________

Teacher ______________________________ Period ____________

Directions: Each student listed below participated in the fall testing. Circle whichever applies from among the following:

E = still enrolled in this class
M = moved to another geometry class in this school
WP = still goes to this school, withdrew passing
WF = still goes to this school, withdrew failing
L = left this school

Note: For each student falling under M, WP, or WF, please indicate the month that student left this class.

		Circle one				Date (if applies)
1.	E	M	WP	WF	L	
2.	E	M	WP	WF	L	
3.	E	M	WP	WF	L	
4.	E	M	WP	WF	L	
5.	E	M	WP	WF	L	
6.	E	M	WP	WF	L	
7.	E	M	WP	WF	L	
8.	E	M	WP	WF	L	
9.	E	M	WP	WF	L	
10.	E	M	WP	WF	L	
11.	E	M	WP	WF	L	
12.	E	M	WP	WF	L	
13.	E	M	WP	WF	L	
14.	E	M	WP	WF	L	
15.	E	M	WP	WF	L	
16.	E	M	WP	WF	L	
17.	E	M	WP	WF	L	
18.	E	M	WP	WF	L	
19.	E	M	WP	WF	L	
20	E	M	WP	WF	L	
21.	E	M	WP	WF	L	
22.	E	M	WP	WF	L	
23.	E	M	WP	WF	L	
24.	E	M	WP	WF	L	
25.	E	M	WP	WF	L	
26.	E	M	WP	WF	L	
27.	E	M	WP	WF	L	
28.	E	M	WP	WF	L	
29.	E	M	WP	WF	L	
30.	E	M	WP	WF	L	

ENROLLMENT INFORMATION FORM - Part II

To the teacher: The purpose of this part is to add the students missed in September, so as to determine precisely how many students are in the population that we are testing.

School ________________________________ Date ____________

Teacher ________________________________ Period ____________

Directions: Please record below the names of all students currently in this class who were not listed in Part I. Indicate why the student missed the September testing.

A = absent during September testing or chose not to take
S = switched from another class in this school
T = transferred into this class from outside the school

Note: Use S for a student who began geometry last year (1979-80) and came into the class at the semester.
For each student falling under S or T, please indicate the month of entry into this class.

Students	Circle one	Date (if applies)
1.	A S T	
2.	A S T	
3.	A S T	
4.	A S T	
5.	A S T	
6.	A S T	
7.	A S T	
8.	A S T	
9.	A S T	
10.	A S T	
11.	A S T	
12.	A S T	
13.	A S T	
14.	A S T	
15.	A S T	
16.	A S T	
17.	A S T	
18.	A S T	
19.	A S T	
20.	A S T	

R

GEOMETRY CONTENT INFORMATION FORM

Teacher's Name ______________________ School __________________________ Date _______

1. Identify below all classes to which this form applies. (Use a separate form for each different type -- non-proof, accelerated, different text, etc. -- of geometry class taught by you.)

   Official name of geometry class _________________________ Period(s) _____

2. In the 1980-81 school year, how many days has this class spent studying geometry? _______

3. Has this course included any geometry proof? (Circle): Yes No

   If Yes, Please go to question 4.
   If No, please go to question 8.

4. On approximately what date did you:

   (a) first introduce the concept of proof? _______________
   (b) first expect students to write their own proofs? _______________

5.A. In this school year, on approximately how many days before today were geometry proofs (either textbook theorems or problems) involved? _______________

B. On those days when geometry proofs were involved, how often were geometry proofs (Circle your best estimate):

N	Virtually never
1/4	About one fourth of the time
1/2	About one half the time
3/4	About three fourths of the time
A	Virtually always

(1) demonstrated to the class by you?	N	1/4	1/2	3/4	A
(2) assigned to students to read and understand?	N	1/4	1/2	3/4	A
(3) assigned to students to do in class or at home?	N	1/4	1/2	3/4	A
(4) demonstrated to the class by students?	N	1/4	1/2	3/4	A
(5) involved in other ways?	N	1/4	1/2	3/4	A

(Explain: _________________________________)

6.A. How many tests and quizzes were entirely on geometry proof?

Number of tests (full period): ______________

Number of quizzes (part of period): _________

B. Estimate the total number of geometry proofs that appeared on all tests and quizzes to date: ___________

7. The columns in the chart below refer to the months of this school year. In each column, put an X in the one row which most nearly describes the emphasis you placed on geometry proof on tests during that month.

Heavy emphasis--Tests and quizzes were all, or almost all, geometry proofs.
Moderate emphasis--About half of each test consisted of geometry proofs.
Light emphasis--There was an average of 1 geometry proof on each test.
No emphasis--There were no geometry proofs on tests.

	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May
Heavy									
Moderate									
Light									
None									

8. Here are some standard geometry theorems. Please circle all letters that apply to the classes covered by this form. Circle ALL appropriate options.

N if the students are not expected to know this theorem.

K if the students are expected to know the statement of this theorem.

A if the students are expected to be able to apply this theorem in proofs.

R if the students are expected to be able to reproduce the textbook proof or some other proof of the theorem.

(a) The base angles of an isosceles triangle are congruent. N K A R

(b) The diagonals of a rectangle are congruent. N K A R

(c) Pythagorean Theorem N K A R

(d) The sum of the measures of the angles of a triangle is 180 (or 180°). N K A R

(e) If two parallel lines are cut by a transversal, alternate interior angles are congruent. N K A R

(h) Two triangles are similar if two angles of one triangle are congruent to two angles of the other. (AA Similarity) N K A R

(i) Two triangles are congruent if three sides of one are congruent respectively to three sides of the other. (SSS Congruence) N K A R

9. Check which, if any, of the following topics occupied at least 5 days of class time thus far this school year:

________ a) parallel lines
________ b) congruent triangles
________ c) quadrilaterals
________ d) similarity
________ e) circles
________ f) area
________ g) coordinate geometry
________ h) constructions
________ i) transformations
________ j) vectors
________ k) surface area or volume
________ l) 3-dimensional geometry (not including surface area and volume)

10. Comments? (Please use the back of the form if necessary).

PROOF INFORMATION FORM

R

Teacher's Name ________________________ School ___________________________ Date__________

1. Identify below all classes to which this form applies. (Use a separate form for each different type -- non-proof, accelerated, different text, etc, -- of geometry class taught by you.)

   Official name of geometry class __________________________ Period(s) ________

2. Here are some problems typically assigned to students in the study of proof. Please circle all letters that apply to this class (or classes):

   N if this problem or a similar problem was not covered

   D if you demonstrated the same or a similar problem

   A if you assigned, either in class or for homework, the same or a similar problem

   T if the same or a similar problem appeared on a test or quiz during the year

   U if you feel it is unfair for these students

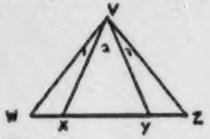
   F if you believe, based upon the instruction the students have received that this is a fair problem for most of the students (i.e., if they learned what was taught, they should be able to do it)

A. Write statements and reasons to complete this proof.

GIVEN: $\angle W \cong \angle Z$

$\overline{WX} \cong \overline{YZ}$

PROVE: $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle W \cong \angle Z$, $\overline{WX} \cong \overline{YZ}$	Given.
2. $\overline{WV} \cong \overline{ZV}$	______________
3. ______________	______________
4. $\angle 1 \cong \angle 3$	______________

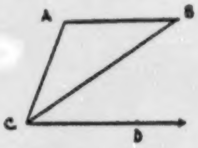
N Not covered
D Demonstrated
A Assigned
T Tested
U Unfair
F Fair

B. Write statements and reasons to complete this proof.

GIVEN: $\overrightarrow{AB} \parallel \overrightarrow{CD}$

$AB = AC$

PROVE: $\overrightarrow{CB}$ bisects $\angle ACD$.



Statements	Reasons
1. $\overrightarrow{AB} \parallel \overrightarrow{CD}$.	Given.
2. $\angle B \cong \angle BCD$.	______________
3. $AB = AC$	Given.
4. ______________	Base angles of an isosceles triangle are congruent (equal in measure).
5. ______________	Transitive property or substitution
6. ______________	Definition of angle bisector

N
D
A
T
U
F

Circle all that apply:

N Not covered
D Demonstrated
A Assigned
T Tested
U Unfair
F Fair

C. **Statement:** If an altitude is drawn to the base of an isosceles triangle, then it bisects the vertex angle.

Suppose you wished to prove the above statement. In the space provided:

1. Draw and label a figure.
2. Write, in terms of your figure, what is given and what is to be proved.

FIGURE: **GIVEN:**

TO PROVE:

D. **Statement:** The diagonals of a rectangle are congruent.

N
D
A
T
U
F

Suppose you wished to prove the above statement. In the space provided:

1. Draw and label a figure.
2. Write, in terms of your figure, what is given and what is to be proved.

FIGURE: **GIVEN:**

TO PROVE:

E. **Statement:** If a line passes through the midpoints of two sides of a triangle, it is parallel to the third side of that triangle.

N
D
A
T
U
F

Suppose you wished to prove the above statement. In the space provided:

1. Draw and label a figure.
2. Write, in terms of your figure, what is given and what is to be proved.

FIGURE: **GIVEN:**

TO PROVE:

Circle all that apply:

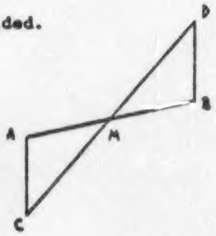
N	Not covered
D	Demonstrated
A	Assigned
T	Tested
U	Unfair
F	Fair

F. Write this proof in the space provided.

GIVEN: M is the midpoint of $\overline{AB}$.

M is the midpoint of $\overline{CD}$.

PROVE: $\triangle ACM \cong \triangle BDM$



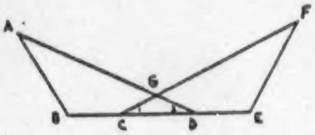
G. Write this proof in the space provided.

GIVEN: $\overline{BD} \cong \overline{EC}$

$\angle 1 \cong \angle 2$

$\angle B \cong \angle E$

PROVE: $\overline{AB} \cong \overline{EF}$



N
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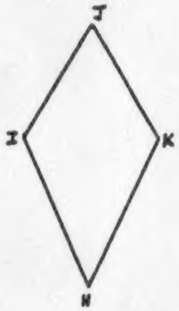
H. Write this proof in the space provided.

GIVEN: Quadrilateral HIJK

HI = HK

IJ = JK

PROVE: $\angle I \cong \angle K$



N
D
A
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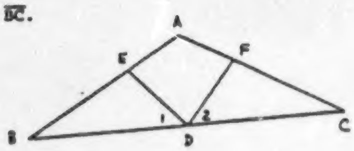
I. Write this proof in the space provided.

GIVEN: D is the midpoint of $\overline{BC}$.

$\angle 1 \cong \angle 2$

$\overline{DE} \cong \overline{DF}$

PROVE: $\triangle ABC$ is isosceles.



N
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A
T
U
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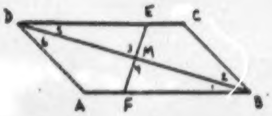
J. Write this proof in the space provided.

GIVEN: AB = DC, AD = BC.

M is the midpoint of $\overline{DB}$.

$\overline{EF}$ contains M.

PROVE: FM = ME



N
D
A
T
U
F

Circle that all apply:

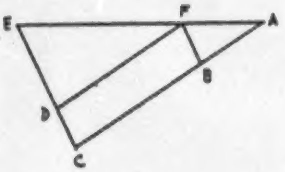
N Not covered
D Demonstrated
A Assigned
T Tested
U Unfair
F Fair

K. Write this proof in the space provided.

GIVEN: $\triangle ABF \sim \triangle ACE$

$\triangle FDE \sim \triangle ACE$

PROVE: BCDF is a-parallelogram.



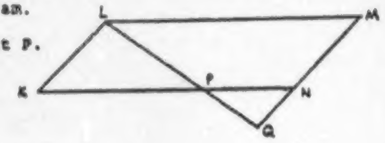
L. Write this proof in the space provided.

GIVEN: KLMN is a parallelogram.

$\overline{LQ}$ and $\overline{KN}$ intersect at P.

N is on line $\overleftrightarrow{MQ}$.

PROVE: $\triangle KLP \sim \triangle NQP$



N
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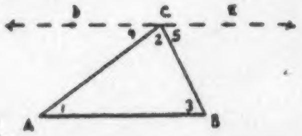
M. This is a theorem you have had. Complete its proof in the space provided.

GIVEN: $\triangle ABC$

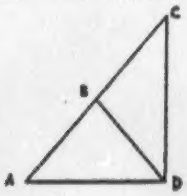
PROVE: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statements	Reasons
1. Through point C draw $\overleftrightarrow{DE}$ so that $\overleftrightarrow{DE} \parallel \overline{AB}$.	1.

N
D
A
T
U
F

N. Write this proof in the space provided.



GIVEN: B is the midpoint of $\overline{AC}$.

AB = BD.

PROVE: $\angle CDA$ is a right angle.

N
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